Part I. Linear Algebra (50%)

1. Compute the following sum of determinants. (15%)

$$\begin{vmatrix} 2 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 6 & 2 & 0 \\ 1 & 1 & -2 & 3 \end{vmatrix} + \begin{vmatrix} 2 & 1 & 2 & 1 \\ 3 & 0 & 1 & 1 \\ -1 & 2 & -2 & 1 \\ -3 & 2 & 3 & 1 \end{vmatrix}.$$

2. Let $\mathbf{v}_1 = \begin{bmatrix} 4 \\ 6 \\ 7 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ and $\mathbf{v}_3 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$, and let $\mathbf{u}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $\mathbf{u}_2 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$ and $\mathbf{u}_3 = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$.
If $\mathbf{x} = 2\mathbf{v}_1 + 3\mathbf{v}_2 - 4\mathbf{v}_3$, determine the coordinates of $\mathbf{x}$ with respect to $[\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3]$. (15%)

3. Eigenvalues play an important role in the solution of system of linear differential equations. Solve the following initial value problem. (20%)

$$y_1'' = 2y_1 + y_2 + y_1' + y_2'$$

$$y_2'' = -5y_1 + 2y_2 + 5y_1' - y_2'$$

$$y_1(0) = y_2(0) = y_1'(0) = 4, \qquad y_2'(0) = -4$$

(背面仍有題目,請繼續作答)

Discrete Mathematics

壹. 是非題 (10%)

1. Any function can be expressed as the sum of an even function and an odd function.
2. $f(x) = x^3 + x + 1$ is reducible in both **R[x] and C[x].**
3. It is necessary to use AND, OR and NOT to construct all Boolean algebra.
4. All prefix codes can be uniquely decoded.
5. The complexity of computing 2-dimensional fast Fourier Transform is $O(N^2)$.
6. Relation $R = \{(1,2),(2,3),(3,4),(4,1)\}$ on set $A = \{1,2,3,4\}$ is transitive.
7. Let $A = (0,1)$. If a sequence is represented by $x_n = 1 - \frac{1}{n}$, then the sequence converges to A.
8. The set of all rational numbers is countable.
9. The generating function of the sequence 1,1,1,…, is $\frac{1}{1-x}$.
10. A function $f(.)$ is called monotonically increasing if $f(x) < f(y)$ for $x < y$.

貳. 選擇題(答案可能不只一個) (15%)

1. Which of the following statements are true?
   a. Any product-of-sum expression can be replaced by a sum-of-product expression to generate the same output.
   b. The number of gate delays of a sum-of-product is 3.
   c. Kruskal's algorithm is usually used to simplify Boolean functions.
   d. $x \oplus y = (x + y)(\overline{xy})$

2. Which of the following statements are true?
   a. $f(x) = x^8 - 1 \in \Re[x]$ has 8 roots.
   b. $f(x) = x^2 + 3x + 2 \in Z_6[x]$ has 2 roots.
   c. The number of polynomials of degree 2 in $Z_3[x]$ is 18.
   d. The degree of the product of two polynomials is equal to the sum of the respective degrees of the two polynomials.

3. Which of the following statements are true?

   a. $\sum_{i=1}^{n} i^2 = (n)(n+1)(n+2)/6$.

   b. $\binom{n+1}{r} = \binom{n}{r} + \binom{n}{r-1}, n > r \geq 0$

   c. Let $F_o = 0$, $F_1 = 1$, and $F_n = F_{n-1} + F_{n-2}$, for $n \geq 2$. Then, $\sum_{i=0}^{k} F_i^2 = F_{k+1} \times F_{k+2}$.

   d. $\sum_{i=1}^{\infty} \frac{1}{i}$ is bounded.

參. 計算題 (Show all detail)

1. Find the homogeneous solution and the non-homogeneous solution of the following recurrence relation. $6a_n - 5 \cdot a_{n-1} + a_{n-2} = \cos(n \cdot \pi)$, where $a_0 = 1$ and $a_{-1} = a_{-2} = 0$, for $n > 0$. **(15%)**
2. What are the overflow conditions of addition or subtraction of two 16-bit binary numbers. 2's complement representation is used in this problem. Prove your answer. **(10%)**