

本試題是否可以使用計算機：可使用，不可使用（請命題老師勾選）

考試日期：0301，節次：3

**Part I. Linear Algebra (50%)**

單選題 (每題 5%)

1. The vectors  $[1 \ 1 \ 1 \ 1]^T$ ,  $[1 \ 0 \ -1 \ 0]^T$ ,  $[1 \ -2 \ -3 \ 4]^T \in R^4$  are
- (A) linearly independent.  
 (B) linearly dependent.  
 (C) mutually orthogonal.  
 (D) able to form a basis for  $R^4$ .  
 (E) able to form a spanning set for  $R^4$ .

2. If  $A = \begin{bmatrix} 4 & 2 & 5 \\ -1 & 0 & 5 \\ 2 & 1 & 2 \end{bmatrix}$ , then

- (A)  $\text{rank}(A) = 2$ .  
 (B)  $\det(A) = 1$ .  
 (C)  $[2 \ 1 \ -2]^T$  is in the nullspace of  $A$ .  
 (D) If  $\mathbf{b} = [0 \ 1 \ 0]^T$ , the linear system  $Ax = \mathbf{b}$  is consistent.  
 (E)  $A$  is row equivalent to  $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ .

3.  $A = \begin{bmatrix} 2 & -3 & 1 \\ 1 & -2 & 1 \\ 1 & -3 & 2 \end{bmatrix}$ . Which of the following is **NOT** true?

- (A) 0 is an eigenvalue of  $e^A$ .  
 (B)  $e$  is an eigenvalue of  $e^A$ .  
 (C)  $A$  is diagonalizable.  
 (D)  $e^A$  is diagonalizable.  
 (E) There exists a  $QR$  factorization for  $e^A$ .

4. Which  $L$  is a linear transformation?

(A)  $L(X) = X + \begin{bmatrix} 1 \\ 4 \end{bmatrix} [1 \ 1 \ 1 \ 1]$ ,  $X \in R^{2 \times 4}$ .

(B)  $L(A) = A - A^T$ ,  $A \in R^{n \times n}$ .

(C)  $L([x_1 \ x_2 \ x_3]^T) = x_1 + \sqrt{2}x_2 - x_3$ .

(D)  $L\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 + x_2 \\ 1 \end{bmatrix}$ .

- (E) None of the above is linear.

(背面仍有題目,請繼續作答)

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5. For any  $f, g \in C[-\pi, \pi]$ , the inner product in the vector space  $C[-\pi, \pi]$  is defined by

$$\langle f, g \rangle = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)g(x)dx.$$

If  $(1 + \cos x) \in C[-\pi, \pi]$ , the length  $\|1 + \cos x\|$  is

- (A)  $\sqrt{2}\pi$  (B)  $\sqrt{3}$  (C)  $\pi$  (D)  $\sqrt{2}$  (E) None of the above is true.

6. Which matrix diagonalizes  $\begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$ ?

- (A)  $\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$  (B)  $\begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}$  (C)  $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$  (D)  $\begin{bmatrix} 1 & 1/\sqrt{2} \\ 0 & -1/\sqrt{2} \end{bmatrix}$  (E) None of the above is true.

7. Which matrix has trace 1 and determinant -2?

- (A)  $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$  (B)  $\begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}$  (C)  $\begin{bmatrix} -1 & \sqrt{3} \\ \sqrt{3} & 1 \end{bmatrix}$  (D)  $\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$  (E) None of the above is true.

8. Let bases  $E = \{v_1, v_2\} = \left\{ \begin{bmatrix} 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$  and  $F = \{u_1, u_2\} = \left\{ \begin{bmatrix} 5 \\ 2 \end{bmatrix}, \begin{bmatrix} 7 \\ 3 \end{bmatrix} \right\}$ .

What is the transition matrix from  $E$  to  $F$ ?

- (A)  $\begin{bmatrix} -9 & 16 \\ -4 & 7 \end{bmatrix}$  (B)  $\begin{bmatrix} -5 & -4 \\ 4 & 3 \end{bmatrix}$  (C)  $\begin{bmatrix} -9 & -4 \\ 16 & 7 \end{bmatrix}$  (D)  $\begin{bmatrix} -5 & 4 \\ -4 & 3 \end{bmatrix}$  (E) None of the above is true.

9.  $L(p(x)) = \begin{bmatrix} \frac{dp(x)}{dx} \Big|_{x=1} \\ \frac{1}{2} \int_0^e p(x) dx \end{bmatrix}$ . If  $p(x) = \alpha x + \beta$ , which matrix  $A$  satisfies  $L(p(x)) = A \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$ ?

- (A)  $A = \begin{bmatrix} 1 & 0 \\ 1 & 1/2 \end{bmatrix}$  (B)  $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$  (C)  $A = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}$  (D)  $A = \begin{bmatrix} 1 & 1/2 \\ 1 & 1 \end{bmatrix}$  (E) No such matrix exists.

10. Let  $\mathbf{x} = [1 \ 0 \ 3 \ 0]^T$ ,  $\mathbf{u}_1 = [1 \ -1 \ 0 \ 1]^T / \sqrt{3}$  and  $\mathbf{u}_2 = [1 \ 1 \ 1 \ 0]^T / \sqrt{3}$ .  $S = \text{span}\{\mathbf{u}_1, \mathbf{u}_2\}$ .

Let  $T$  be the orthogonal complement of  $S$ ,  $T = S^\perp$ . The vector in  $T$  that is closest to  $\mathbf{x}$  is

- (A)  $(1/3)[5 \ 3 \ 4 \ 1]^T$  (B)  $(1/3)[-2 \ -3 \ 5 \ -1]^T$  (C)  $(-2/3)[0 \ -2 \ -1 \ -2]^T$  (D)  $(1/6)[3 \ -11 \ 4 \ 5]^T$  (E) None of the above is true.

## Part II. Discrete Mathematics 2008 (50%)

1. [10%]

(1) (5%) If statement  $q$  has the truth value 1, determine all truth value assignments for the primitive statements,  $p$ ,  $r$ , and  $s$  for which the truth value of statement

$$(q \rightarrow [(\neg p \vee r) \wedge \neg s]) \wedge [\neg s \rightarrow (\neg r \wedge q)]$$

is 1.

(2) (5%) Express the negation of the statement  $p \leftrightarrow q$  in term of the connectives  $\wedge$  and  $\vee$ .

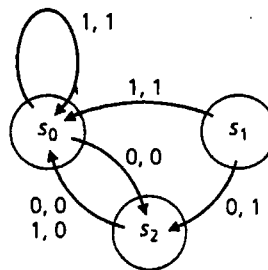
2. [15%]

(1) (5%) Determine which of the following statements are true and which are false.

- (a)  $Z^+ \subseteq Q^+$
- (b)  $R^+ \cap C = R^+$
- (c)  $R^+ \subseteq Q$
- (d)  $Q^+ \cap Z = Z$
- (e)  $Z^+ \cup R^+ = R^+$

(2) (10%) Prove that for all  $n \in Z^+, n > 3 \Rightarrow 2^n < n!$

3. [15%] Let  $M$  be the finite state machine in the following figure. For states  $S_i, S_j$ , where  $0 \leq i, j \leq 2$ , let  $O_{ij}$  denote the set of all nonempty output strings that  $M$  can produce as it goes from state  $S_i, S_j$ , e.g.,  $O_{20} = \{0\} \{1, 00\}^*$ . Find  $O_{22}, O_{11}$ , and  $O_{10}$ .



4. [10%] Find the coefficient of  $x^{50}$  in  $(x^7 + x^8 + x^9 + \dots)^6$ .