※ 考生請注意：本試題 □可　■不可 使用計算機

Part I. Linear Algebra (50%)

簡答題：(1-7 題每題 6 分, 第 8 題 8 分)

1. $L(\mathbf{x}) = \begin{bmatrix} x_1 - x_2 \\ x_2 - x_3 \end{bmatrix}$, $\mathbf{x} \in R^3$. Find ker(L).

2. $L: R^3 \rightarrow R^2$. $L(\mathbf{x}) = x_1\mathbf{u}_1 + (x_2+x_3)\mathbf{u}_2$, where $\mathbf{u}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\mathbf{u}_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$. Find the matrix A representing L w. r. t. the bases $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ and $\{\mathbf{u}_1, \mathbf{u}_2\}$.

3. $S = \mathrm{span}(\mathbf{v}_1, \mathbf{v}_2)$, $\mathbf{v}_1 = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 6 \\ 3 \\ 3 \end{bmatrix}$. Find the vector $\mathbf{p}$ in S that is closest to $\mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$.

4. Let S be the subspace of R^4 spanned by $[1 \ 0 \ -2 \ 1]^T$, $[0 \ 1 \ 3 \ -2]^T$, and $[2 \ -3 \ 1 \ 0]^T$. Find an orthonormal basis for $S^\perp$.

5. $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. Find trace (e^A).

6. $A, B \in R^{n \times n}$, $A\mathbf{x} = B\mathbf{x}$ for all $\mathbf{x} \in R^n$. Find the nullity of $(A-B)$.

7. $A = \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 0 & -2 \\ -6 & 1 \end{bmatrix}$. Solve the matrix equation $AX+2B=C$, where $X \in R^{2 \times 2}$.

8. Diagonalize $A = \begin{bmatrix} 1 & -3 & 2 \\ 1 & -2 & 1 \\ 2 & -3 & 1 \end{bmatrix}$ to be a diagonal matrix D. What is det(D)? (8%)

(背面仍有題目,請繼續作答)

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Part II. Discrete Mathematics 2009 (50%)

1. Determine the following values. (每題 5 分)

   (a) the number of permutations of the digits 0,1,2,3,4,5 either start with a 3 or end with a 2.

   (b) the number of distinct terms are there in the expansion of $(a+3b-2c+1)^6$.

   (c) the number of integer solutions there are to $x_1 + x_2 + x_3 + x_4 = 19$, $0 \le x_i < 8$ for all $i=1\sim4$.

   (d) the number of closed binary operations on A have c as the identity, where A={a, b, c, d, e}.

2. 計算題 （每題 10 分）

   (a) Determine the sequence generated by the generating function $f(x) = \dfrac{1}{(3-x)} - (3-x)$.

   (b) If a fair die is rolled 12 times, what is the probability that the sum of the rolls is 24?

   (c) Solve the recurrence relation $a_n = 6a_{n-1} - 9a_{n-2}$, $n \ge 2$, $a_0=5$, $a_1=12$.