系所組別： 資訊工程學系
考試科目： 計算機數學

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Part I.

Linear Algebra (50%)

1. 單選題（15 小題共 50%，1.1-1.10 每題 3%，1.11-1.15 每題 4%。）
（注意：請將答案標上題號依序寫在答案卷上，而非此試題紙）

1.1) A, B and C are $n \times n$ matrices. Which of the following is correct?

(A) $(A+B)^2 = A^2 + 2AB + B^2$

(B) If $AB = AC$ and $A \neq O$ (zero matrix), then $B = C$.

(C) Suppose that E_1, E_2, and E_3 are (row) elementary matrices such that $E_3E_2E_1A = U$, and U is an upper triangular matrix. Let $L = E_1^{-1}E_2^{-1}E_3^{-1}$, then L is a lower triangular matrix.

(D) Matrix A is singular if and only if the reduced row echelon form of A is I.

(E) None of the above.

1.2) Which of the following is correct?

(A) $(x^2 - 2x + 1)$ and $|x - 1|$ are linearly dependent in the vector space $C[0, 2]$. (Note: $C[a, b]$ denotes the set of all real-valued functions that are defined and continuous on the closed interval $[a, b]$.)

(B) $\cos(x)$, 1, $\sin(x)$ are linearly independent in $C[-\pi, \pi]$.

(C) $A = \begin{bmatrix} \cos(\pi/4) & \sin(\pi/4) \\ -\sin(\pi/4) & \cos(\pi/4) \end{bmatrix}, B = \begin{bmatrix} e & 1 \\ 1 & e^{-1} \end{bmatrix}, C = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}, D = \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix}, E = \begin{bmatrix} 0 & 8 \\ 4 & 0 \end{bmatrix}$ are linearly independent in $R^{2\times2}$

(D) $\mathbf{v}_1 = [1\ 8\ 9\ 5]^T$, $\mathbf{v}_2 = [1\ 7\ 8\ 9]^T$, and $\mathbf{v}_3 = [3\ 0\ 0\ 1]^T$ form a spanning set for R^4

(E) None of the above.

1.3) Which of the following statements about subspaces is correct?

(A) $S = \{\mathbf{x} \mid \mathbf{x} = [1\quad x_2\quad x_3]^T, \mathbf{x} \in R^3\}$ is a subspace of R^3

(B) $S = \{\mathbf{x} \mid \mathbf{x}^T\mathbf{x} = 1, \mathbf{x} \in R^3\}$ is a subspace of R^3

(C) $S = \{A \mid A$ is a triangular matrix in $R^{3\times3}\}$ is a subspace of $R^{3\times3}$

(D) For a singular 3×3 matrix A, $S = \{\mathbf{x} \mid A\mathbf{x} = \mathbf{0}, \mathbf{x} \in R^3\}$ is a subspace of R^3

(E) None of the above.

1.4) Let A be an $n \times n$ matrix. Which of the following about determinants is NOT correct?

(A) $\det(3A) = 3^n\det(A)$

(B) $\det(\text{adj } A) = \det(A)^{n-1}$

(C) $\det(A^T) = \det(A)$

(D) If matrix B is row equivalent to A, then $\det(A) = \det(B)$.

（背面仍有題目,請繼續作答）

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1.5) Which of the following statements is NOT correct?

(A) If $A\mathbf{x} = \mathbf{b}$ is consistent, then $\mathbf{b}$ is in the row space of A.

(B) If $\mathbf{x}$ is a nonzero vector in R^n and satisfies $A\mathbf{x} = \mathbf{0}$, then $\det(A) = 0$.

(C) Let L: $V \to V$ be a linear transformation. If $\mathbf{x}$ is a vector in the kernel of V, then $L(\mathbf{v} + \mathbf{x}) = L(\mathbf{v})$ for all $\mathbf{v} \in V$.

(D) If $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n$ span R^n, then $\{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n\}$ is a basis for R^n

1.6) A is an $m \times n$ matrix ($m \neq n$). Which of the following is NOT correct?

(A) If the row vectors of A are linearly independent, the nullspace $N(A) = \{\mathbf{0}\}$.

(B) Suppose that $A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$ and A_{11} is a nonsingular $k \times k$ matrix. If $A = \begin{bmatrix} I & O \\ B & I \end{bmatrix} \begin{bmatrix} A_{11} & A_{12} \\ O & C \end{bmatrix}$, then $C = A_{22} - A_{21}A_{11}^{-1}A_{12}$.

(C) If A has a right inverse C such that $AC = I_m$, then the column vectors of A span R^m

(D) If A has a left inverse B such that $BA = I_n$, then the column vectors of A are linearly independent.

1.7) Which of the following operators is a linear transformation?

(A) An operator L: $R^3 \to R^3$ defined by $L(\mathbf{x}) = \mathbf{x} + \mathbf{a}$, where $\mathbf{a}$ is a constant vector in R^3

(B) An operator L: $R^2 \to R^3$ defined by $L(\mathbf{x}) = [(x_1+x_2)^2 \quad x_1+x_2 \quad x_2]^T$

(C) An operator L defined on P_3: $L(p(x)) = p(x) - p'(x)$.

(D) An operator L defined on $R^{n\times n}$: $L(A) = e^A$

(E) None of the above.

1.8) Let X be the subspace of R^4 spanned by $\mathbf{e}_1$ and $\mathbf{e}_2$, and Y be the subspace spanned by $\mathbf{e}_3$. Which of the following is correct? ($\mathbf{e}_1 = [1 \quad 0 \quad 0 \quad 0]^T$, $\mathbf{e}_2 = [0 \quad 1 \quad 0 \quad 0]^T$, $\mathbf{e}_3 = [0 \quad 0 \quad 1 \quad 0]^T$)

(A) X and Y are orthogonal, $X \perp Y$.

(B) X and Y are orthogonal complements, $X = Y^{\perp}$

(C) R^4 is a direct sum of X and Y, $R^4 = X \oplus Y$

(D) The union of X and Y, $X \cup Y$, is also a subspace of R^4

(E) None of the above.

1.9) If A is a 2×2 matrix with trace 101 and determinant 100, then

(A) the characteristic polynomial of A is $\lambda^2 + 101\lambda + 100$.

(B) both A and e^A can be diagonalized to $\begin{bmatrix} 1 & 0 \\ 0 & 100 \end{bmatrix}$.

(C) A^{-1} has an eigenvalue 0.01.

(D) A^n has a trace $10^{2n} + 1$ and a determinant $(-1)^n 10^{2n}$

(E) none of the above is correct.

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1.10) The real action of $A\mathbf{x} = \mathbf{b}$ is a transformation from a vector $\mathbf{x}$ in the row space, $R(A^T)$, to a vector $\mathbf{b}$ in the column space, $R(A)$. If $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 5 & 0 \end{bmatrix}$, then the inverse transformation, from $R(A)$ to $R(A^T)$, will map a vector $\mathbf{b} = [5 \quad 3]^T$ to

(A) $\mathbf{x} = [1 \quad 1 \quad 2]^T$ (B) $\mathbf{x} = [1 \quad 1]^T$ (C) $\mathbf{x} = [5/3 \quad 3/5 \quad 0]^T$ (D) $\mathbf{x} = [5/3 \quad 3/5]^T$ (E) none of the above.

1.11) Use a quadratic curve that passes the origin to fit points (0, 1), (1, −1), and (2, 2) based on the least-squares criterion. The best-fit curve will be

(A) $y = -x^2 + 2x$ (B) $y = x^2 + 4x$ (C) $y = 0.5x^2 - 1.5x$ (D) $y = 2x^2 - 3x$ (E) none of the above.

1.12) In the vector space $C[-1, 1]$, an inner product is defined by $< f, g >= \frac{1}{2}\int_{-1}^{1} f(x)g(x)dx$ Given $f(x) = 3x + 1 \in C[-1,1]$, the value of $\|f(x)\|$ is

(A) 8 (B) $3\sqrt{2}$ (C) $2\sqrt{2}$ (D) 2 (E) none of the above.

1.13) $A = \begin{bmatrix} 4 & 6 & -2 \\ -1 & -1 & 1 \\ 0 & 0 & c \end{bmatrix}$ is not diagonalizable if

(A) $c = 0$ (B) $c = \pm 1$ (C) $c = 2$ (D) $c = 3$ (E) none of the above.

1.14) Consider a vector space V with an ordered basis $\{\mathbf{v}_1, \mathbf{v}_2\}$. The vectors in V are expressed in terms of $\mathbf{v}_1$ and $\mathbf{v}_2$. A linear mapping L . $R^3 \to V$ is defined by $L(\mathbf{x}) = (x_1 + 5x_2 + 4x_3)\mathbf{v}_1 + (x_1 + 2x_2 + 3x_3)\mathbf{v}_2$. Given a subspace $S = \text{span}\{[1 \quad 1 \quad 1]^T\}$ in R^3, what is the image of S, $L(S)$?

(A) Span$\{[1 \quad 0 \quad 4]^T\}$ (B) Span$\{[5 \quad 3]^T\}$ (C) $[0 \quad 0]^T$ (D) $[10 \quad 6]^T$ (E) None of the above.

1.15) Let $A = \begin{bmatrix} 1 & -2 & -1 \\ 2 & 0 & 1 \\ 2 & -4 & 2 \\ 4 & 0 & 0 \end{bmatrix}$. Use the Gram-Schmidt process to find an orthonormal basis for the column space of A. Which of the following vectors is NOT in this basis?

(A) $\left[\frac{1}{5} \quad \frac{2}{5} \quad \frac{2}{5} \quad \frac{4}{5}\right]^T$ (B) $\left[\frac{2}{5} \quad -\frac{1}{5} \quad -\frac{1}{5} \quad \frac{2}{5}\right]^T$ (C) $\left[-\frac{2}{5} \quad \frac{1}{5} \quad -\frac{4}{5} \quad \frac{2}{5}\right]^T$ (D) $\left[-\frac{4}{5} \quad \frac{2}{5} \quad \frac{2}{5} \quad -\frac{1}{5}\right]^T$ (E) All of the above are in this basis.

(背面仍有題目,請繼續作答)

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Part II.

Discrete Mathematics (50%)

1 單選題
(注意：請將答案標上題號依序寫在答案卷上，而非此試題紙)

1.1) [5%] Which of the following statement is NOT true?
(A) $\phi \subseteq \{\phi\}$
(B) $\phi \subseteq \phi$
(C) $\phi \subset \{\phi\}$
(D) $\phi \subset \phi$
(E) $\phi \in \{\phi\}$

1.2) [5%] Let A={a, b, c, d} and B ={1, 2, 3, 4}, which of the following statement is NOT true?
(A) There are 4^{10} closed binary operations on A that have an identity.
(B) There are 2^{16} relations from A to B.
(C) There are 4! one-to-one functions from A to B.
(D) There are 4! onto functions from A to B.
(E) There are 4^{6} closed binary operations on A that are commutative.

1.3) [5%] Which of the following statement is true?
(A) $A\Delta(B \cap C) = (A\Delta B) \cap (A\Delta C)$, Δ : *symmetric difference*.
(B) $(A - B) \cup (A \cap B) = \overline{A}$
(C) Negation of $\exists x[(p(x) \vee q(x)) \rightarrow r(x)]$ is $\forall x[(p(x) \vee q(x)) \wedge \neg r(x)]$
(D) If A={01}, B={01, 000, 0111}, we can say AB={01, 000, 0101, 0111, 01000, 010111}.
(E) None of the above

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1.4) [10%]Which of the following statement is true?

(A) The sum of all the coefficients in the expansions of $(x-3y+3)^{10}$ is $(-2)^{10}$

(B) Consider the 2^{19} compositions of 20, the number of compositions in which each summand even is 2^{10}

(C) The number of derangements of 1, 2, 3, 4, 5, 6, 7, 8 start with 5, 6, 7, and 8, in some order is 9^2

(D) If n is formed by using the digits 3, 3, 4, 5, 5, 6, 7. The number of positive integers n that exceed 4,000,000 is 720.

(E) None of the above.

2 計算題

2.1) [15%] In a social network, we want to match each of four women with one of five men. According to the information they provided, we can draw the following conclusions.

- Woman 1 would not be compatible with man 1, 3, or 5.
- Woman 2 would not be compatible with man 2, or 4.
- Woman 3 would not be compatible with man 3, or 5.
- Woman 4 would not be compatible with man 4.

In how many ways can the service successfully match each of the four women with a compatible partner?

2.2) [10%] (a) Find the exponential generating function for the number of ways to arrange n letters, $n \geq 0$, selected from the word "MISSISSIPPI" (b) in (a), what is the exponential generating function if the arrangement must contain at least two I's.