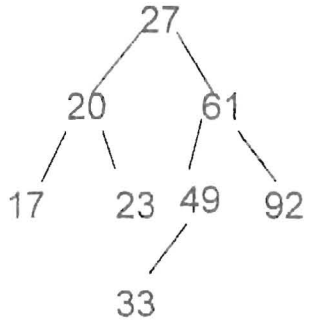
※ 考生請注意：本試題 □可 ☑不可 使用計算機

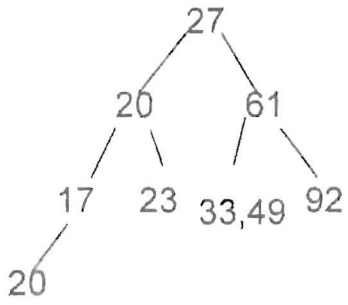
一、Data Structures (50%)

1. (15%)

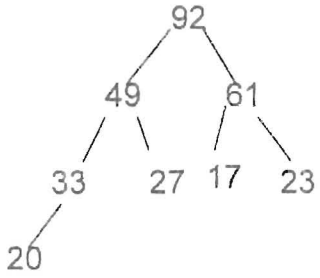
Insert a sequence of keys (27, 49, 17, 20, 61, 23, 92, 33) into a data structure that has no keys in the beginning. The results are depicted as follows. Please answer the corresponding data structure for (a), (b), and (c), respectively.



(a)



(b)



(c)

2. (25%)

We know the red-black tree is one of many search tree schemes that are approximately balanced in order to guarantee that basic dynamic set operations take O(*lg n*) time in the worst case.

(a) (15%) Please augment the red-black tree to maintain a dynamic set Q of numbers that supports the operation MIN-GAP, which gives the magnitude of the difference of the two closest numbers in Q. For example, if Q={1, 3, 9, 15, 18, 22}, then MIN-GAP(Q) returns 2, since 1 and 3 are the two closest numbers in Q. Make the operations MIN-GAP as efficient as O(*lg n*) where n is the size of Q.

(b) (10%) According to the augmented red-black tree, it is also possible to simultaneously find the two closest numbers that result in the minimum gap in O(*lg n*) time. Please explain how to do it?

(背面仍有題目,請繼續作答)

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3. (10%) For the following applications (a)-(f), please choose the most suitable data structure or algorithm from the candidates (1)-(9) to handle them. We denote $N(j)=i$ if application (j) is matched with data structure or algorithm (i) where $j \in \{a, b, c, d, e, f\}$ and $i \in \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Please find the value $\sum_{A=a}^{A=f} (N(A))^2 = ?$

(a) A network routing protocol in which the routing algorithm must determine a minimal spanning tree.
(b) The process scheduler in an operation system that needs to dispatch processes in the order of arrival.
(c) The dictionary look-up function that needs the index of words.
(d) The maze problem that needs to keep track of the path visited.
(e) The database query optimizer that needs the index of data records.
(f) The database system that is used in a hospital in which the data of patients accessed will be accessed again frequently.

(1) Stack
(2) Queue
(3) Splay tree
(4) B-tree
(5) Red-Black tree
(6) Kruskal's algorithm
(7) Dijkstra's algorithm
(8) Topological sorting
(9) Hashing

二、Algorithms (50%)

4. (20%) Solving the recurrence $T(n) = 2T(n/2) + n/\log_2 n$ using Θ notation.

5. (10%) What is an optimal **Huffman code** for the following set of frequencies: a: 25, b: 3, c: 12, d: 16, e: 39, f: 5 , g: 13.

6. (10%) Give a $O(V^2 \lg V + VE)$-time algorithm to find the shortest paths between all pairs of vertices in a weighted and directed graph $G=(V, E)$.

7. (10%) Describe an algorithm that, given n integers in the range 0 to k, preprocesses its input and then answers any query about how many of the n integers fall into a range $[a...b]$ in $O(1)$ time, where $a, b \in \{0,1,...,k\}$. Your algorithm should use $\Theta(n+k)$ preprocessing time.