※ 考生請注意：本試題 ☑可 ☐不可 使用計算機

1. [10%] Find the probability of randomly selecting 3 good apples in succession from a basket containing 10 apples of which 4 have spoiled.

2. [15%] A coin is tossed twice. Let X denote the number of heads on the first toss and Y the total number of heads on the 2 tosses. If the coin is unbalanced and a head has 40% chance of occurring, find
   (a) the joint probability distribution of X and Y;
   (b) the marginal distributions of Y;
   (c) the probability that at least 1 head occurs.

3. [10%] Prove that the variance of a random variable X is $\sigma^2 = E(X^2) - \mu^2$

4. [10%] An electrical firm manufactures a 100-watt light bulb, which, according to specifications written on the package, has a mean life of 900 hours with a standard deviation of 50 hours. At most, what percentage of the bulbs fail to last even 700 hours? Assume that the distribution is symmetric about the mean. [Hint: use Chebyshev's theorem]

5. An experiment often consists of repeated trials, each with two possible outcomes that may be labeled success or failure. The process is referred to as a Bernoulli process. Each trial is called a Bernoulli trial.
   [10%] (a) Please describe four properties of the Bernoulli process.
   [5%] (b) Please write out the formula of the distribution $b(x; n, p)$ of **binomial random variable** X, the number of successes in n independent trials. A Bernoulli trial can result in a success with probability p and a failure with probability $q = 1 - p$.
   [10%] (c) Please use binomial expansion of $(q + p)^n$ to show that $\sum_{x=0}^{n} b(x; n, p) = 1$.

6. [10%] Let X be a binomial random variable with probability distribution $b(x; n, p)$. Prove that Poisson distribution is a limiting form of the binomial distribution, i.e.,
   when $n \to \infty, p \to 0$, and $\mu = np$ remains constant, $b(x; n, p) \to p(x; \mu)$.

7. [20%] Find the moment-generating function of the **binomial random variable** X and then use it to verify that $\mu = np$ and $\sigma^2 = npq$.