編號： 286　系所：醫學資訊研究所　　科目：機率統計

本試題是否可以使用計算機：☐可使用，☑不可使用　（請命題老師勾選）

Probability & Statistics 2007 (100%)

1. [15%] A system consists of three components, each having a probability $p = 0.1$ of failing. The cost of repairing a failed component is \$100. The system is down if two or more components fail and the cost of down time during the repairs is \$1000 in addition to the cost of repairing the failed components. Find the expected value of the cost of failures.

2. [20%] Prove **Chebyshev's theorem**: the probability that any random variable X will assume a value within k standard deviation of the mean is at least $1-1/k^2$, i.e.,

   $$P(\mu - k\sigma < X < \mu + k\sigma) \geq 1 - \frac{1}{k^2}.$$

3. [15%] Five new jobs are opening up at a computer manufacturing plant, but 100 applicants show up for the positions. To select the best five from among the applicants, the company gives a test that covers hardware skill, software programming, and mathematical ability. The mean grade on this test turns out to be 60, and the scores have a standard deviation 5. Can a person who has a 90 score count on getting one of the jobs? Assume that the distribution is symmetric about the mean. [Hint: use Chebyshev's theorem]

4. [20%] Prove that the mean and variance of the Poisson distribution $p(x;\lambda t) = \frac{e^{-\lambda t}(\lambda t)^x}{x!}$ both have the value λt.

5. [15%] Please simply illustrate gamma distributions, $f(x) = \begin{cases} \frac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta}, & x > 0 \\ 0, & \text{elsewhere,} \end{cases}$

   where $\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx$, $\alpha > 0$, and $\beta > 0$, with a diagram in the three cases: $\alpha = 1$, $\beta = 1$; $\alpha = 2$, $\beta = 1$; and $\alpha = 4$, $\beta = 1$.

6. [15%] Let a random variable X with the expected value (mean) μ. Which distribution has a larger variance in the following figure? How to make the variance of a distribution smaller?