

系所組別：醫學資訊研究所

考試科目：程式設計

考試日期：0307·節次：2

※ 考生請注意：本試題 可 不可 使用計算機

## 一、 Data Structures (50%)

1. (15%) The Fibonacci polynomials are defined by the recurrence relation

$$F_n(X) = X \cdot F_{n-1}(X) + F_{n-2}(X) \text{ where } F_0(X) = 1, F_1(X) = X, \text{ and } n \geq 2.$$

How many memory spaces are actually needed to hold the Fibonacci polynomials  $F_0, F_1, \dots, F_{100}$ ?

(a) below 4000 (b) 4000~4500 (c) 4501~5000 (d) 5001~5500 (e) above 5500

2. (10%) Which of the following statements are true.

(a) In a directed graph  $G$ , if vertex  $x$  has both incoming and outgoing edges, its tree in the DFS forest contains more than one vertex.(b) A  $d$ -ary heap is like a binary heap, but non-leaf nodes have  $d$  children instead of 2 children. The running time of the efficient implementation of Extract-Max in a  $d$ -ary max-heap with  $n$  elements is  $\Theta(\log_d n)$ .(c) A Hamiltonian Path in graph  $G$  passes through each node  $v \in V$  exactly once. Given a directed acyclic graph  $G=(V, E)$ , its Hamiltonian path  $v_1, v_2, \dots, v_n$  must be a topological ordering of  $G$ .(d) In an undirected graph  $G$ , if there is a path between two vertices  $x$  and  $y$  then in the DFS tree of  $G$ , either  $x$  is a descendant of  $y$  or  $y$  is a descendant of  $x$ .

3. (25%)

(a) An edge,  $(u, v)$ , of a connected graph,  $G$ , is a bridge iff its deletion from  $G$  produces a graph that is no longer connected. Please directly modify the function `bicon` in the following code segment to find the bridges in a graph. (20%)

(b) What is the time complexity of the new function? (5%)

(背面仍有題目,請繼續作答)

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Code segment for Problem 3 of Data structure

```
#define MIN2(x,y) ((x) < (y) ? (x):(y))
short int dfn[MAX_VERTICES];
short int low[MAX_VERTICES];
void bicon(int u, int v)
/* Compute dfn and low, and output the edges of G by their biconnected
components, v is the parent (if any) of u in the resulting spanning tree. It is assumed
that all entries of dfn[] have been initialized to -1, num is initially to 0, and the stack is
initially empty */
    node_pointer ptr;
    int w,x,y;
    dfn[u] = low[u] = num++;
    for (ptr = graph[u]; ptr; ptr = ptr->link) {
        w = ptr->vertex;
        if (v!=w && dfn[w] < dfn[u])
            push(u,w);
        if (dfn[w] < 0) {
            bicon(w,u);
            low[u] = MIN2(low[u],low[w]);
            if (low[w] >= dfn[u])
                printf("New biconnected component:");
            do{
                pop(&x,&y);
                printf("<%d,%d>",x,y);
            } while (!((x == u) && (y == w)));
            printf("\n");
        }
        else if (w != v)
            low[u] = MIN2(low[u],dfn[w]);
    }
}
```

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## 二、 Algorithms (50%)

4. (10%) Solving the recurrence  $T(n) = T(n-1) + \frac{1}{n}$  using  $\Theta$  notation.
5. (20%) Given two sequences  $X = \langle x_1, x_2, \dots, x_m \rangle$  and  $Y = \langle y_1, y_2, \dots, y_n \rangle$ , define  $c[i, j]$  to be the length of an LCS (longest common subsequence) of the sequences  $X_i = \langle x_1, x_2, \dots, x_i \rangle$  and  $Y_j = \langle y_1, y_2, \dots, y_j \rangle$ . Write the recursive formula to compute  $c[i, j]$ .
6. (10%) Prove or disprove: The single-source shortest paths problem can be solved in linear time in directed acyclic graphs.
7. (10%) Express the function  $\frac{n^3}{1000} - 100n^2 - 100n + 3$  in terms of  $\Theta$ -notation.