※ 考生請注意：本試題 ☑可 □不可 使用計算機

1. [10%] Please describe Bayes' Rule and give a simple proof.

2. [10%] An allergist claims that 60% of the patients she tests are allergic to some type of weed. What is the probability that (a) exactly 3 of her next 4 patients are allergic to weeds? (b) none of her next 4 patients are allergic to weeds?

3. [10%] Let X and Y be two random variables with joint probability distribution $f(x, y)$ and marginal distributions $g(x)$ and $h(y)$, respectively. Please show that the random variables X and Y are said to be statistically independent if and only if $f(x, y) = g(x)h(y)$ for all (x, y) within their range.

4. [30%] Consider the random variables X and Y with joint density function
$$f(x, y) = \begin{cases} x + y, & 0 \le x, y \le 1 \\ 0, & \text{elsewhere.} \end{cases}$$
   (a) Find the marginal distributions of X and Y
   (b) Are X and Y independent?
   (c) Find $P(X > 0.5, Y > 0.5)$.
   (d) Find the covariance of X and Y.

5. [10%] Show that Covariance$(aX, bY) = ab$ Covariance(X, Y).

6. [10%] Prove that the mean and variance of the Poisson distribution $p(x; \lambda t) = \dfrac{e^{-\lambda t}(\lambda t)^x}{x!}$ both have the value λt.

7. [10%] Suppose the probability is 0.2 that any given person will believe a tale about the marriage of a famous actress. What is the probability that
   (a) the fifth person to hear this tale is the fourth one to believe it?
   (b) the third person to hear this tale is the first one to believe it?

8. [10%] Figure 8.11 shows the relationship between a standard normal distribution ($v = \infty$) and t-distribution with 2 and 5 degrees of freedom. Please write out the values of the freedom degree a, b, c with 2, 5, ∞.
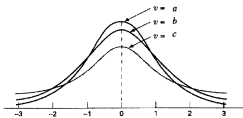

Figure 8.11 The *t*-distribution curves for $v = 2, 5,$ and ∞.