系所組別: 地球科學系甲、乙組

考試科目: 微積分

※ 考生請注意:本試題 □可 ☑不可 使用計算機

1. (10) Show that the equation $x^3 + x - 1 = 0$ has only one real root.

2. (10) Evaluate the indefinite integral
$$\int \frac{1+x}{1+x^2} dx$$

3. (10) Find the area of the largest rectangle that can be inscribed in a semicircle of radius r.

4. (10) Prove the identity: $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$

5. (10) Find the limit
$$\lim_{x \to a^+} \frac{\cos x [\ln(x-a)]}{\ln(e^x - e^a)}$$

6. (10) Find the volume common to two shpere, each with radius r, if the center of each sphere lies on the surface of the other sphere.

7. (10) Find the arc length of the cardioid $r = 1 + \sin\theta$

8. (10) Evaluate $\int\int_D xy \, dA$, where D is the region bounded by the line $y = x - 1$ and the parabola $y^2 = 2x + 6$.

9. (10) Find the Taylor series expansion of $\sin x$ and determine the radius of convergence.

10. (10) Find the radius of convergence and interval of convergence for the series:

(a) (5)
$$\sum_{n=0}^{\infty} \frac{(x-2)^n}{n^2+1}$$

(b) (5)
$$\sum_{n=1}^{\infty} n!(2x-1)^n$$