系所組別：地球科學系甲、乙組
考試科目：應用數學
考試日期：0223，節次：4

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (25 %) Please derive that the gradient of a scalar, f, and divergence and curl of a vector, **A**, in a spherical coordinate (**r** , θ, φ), where r is radius, θ is zenith angle and φ is azimuthal angle, can be stated as follows.

$$\nabla f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \varphi}\hat{\varphi}$$

$$\nabla \cdot A = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta A_\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \varphi}(A_\varphi)$$

$$\nabla \times \mathbf{A} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial \theta}(\sin\theta A_\varphi - \frac{\partial A_\theta}{\partial \varphi})\right]\hat{r} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial \varphi} - \frac{\partial}{\partial r}(rA_\varphi)\right]\hat{\theta} + \frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial \theta}\right]\hat{\varphi}$$

2. (15 %) Find the Fourier series of a periodic step function as follows,

$$f(x) = \begin{cases} -k & if -2 < x < 0 \\ k & if\ 0 < x < 2 \end{cases}$$

where the period equals to 4

3. (15 %) Find the Fourier integral of a pulse function as follows

$$f(x) = \begin{cases} 1 & if\ |x| < 1 \\ 0 & if\ |x| > 1 \end{cases}$$

4. Find eigenvalues and corresponding eigenvectors of the following matrices.

a. (5 %)

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

b. (10 %)

$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 2 & 0 & 3 & 0 \\ 1 & 4 & 2 & -6 \end{bmatrix}$$

(背面仍有題目,請繼續作答)

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5. Please find general solutions of the following differential equations.

a. (10 %)

$$y'' + y = tanx$$

b. (10 %)

$$x^2y'' - 7xy' + 16y = 0$$

c. (10%)

$$W'' + \frac{1}{r} W' + k^2W = 0$$

where k is a constant and W(r) has the boundary condition of W(R) = 0.