系所組別：地球科學系甲、乙組

考試科目：應用數學　　　　　　　　　　　　　　　　　　考試日期：0212，節次：4

第 1 頁，共 2 頁

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Find eigenvalues and eigenvector of the following matrices.

   (a) $\begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}$ (5%)

   (b) $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ (5%)

   (c) Please describe the physical meaning of eigenvalue and eigenvector (you could use an example to illustrate them). (10%)

2. Suppose you have a forced mass-spring system that could be described as,

   $$my'' + cy' + ky = f_0 sinwt$$

   where m is mass of the spring, c is the damping constant, k is the spring constant and $f_0 sinwt$ is the periodic external force.　Please describe the system in detailed (mathematical illustration plus interpretation in physics) when

   　　　(a) c = 0 (10%)

   　　　(b) c ≠ 0 (15%)

3. Suppose a single pulse function, $f(x) = \begin{cases} 1 & if \ |x| < 1 \\ 0 & if \ |x| > 1 \end{cases}$
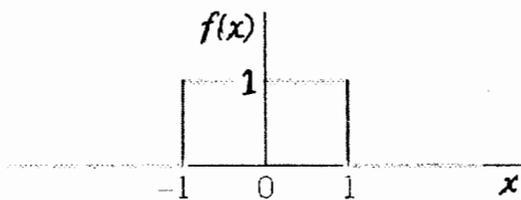
   (a) Please express the single pulse function in the form of Fourier integral. (8%)

   (b) The Fourier integral of such pulse function often lead to sine integral, please evaluate the integral of

   $$\int_0^\infty \frac{sin w}{w} dw$$ (7%)

   (c) Please describe the Gibb's phenomenon. (5 %)

4. Find the general solutions of the following ODEs

(a) $y'' - 4y' + 4y = x^2 e^x$　　　　　　　　　　　(7%)

(b) $y'' + 9y = \cos x + \frac{1}{3}\cos 3x$　　　　　　　(6%)

(c) $y'' + 2y' + y = 2x\sin 2x$　　　　　　　　　(7%)

5. Please evaluate the following integrals:

(a) $\int x e^x dx$　　　　　　　(5%)

(b) $\int \frac{x}{\sqrt{1+x^2}} dx$　　　　　　　(5%)

(c) $\int \sin(3x)\cos(2x)dx$　　　　(5%)