

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Find the eigen values and corresponding eigenvectors of the following matrices,

(a) $\begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$ (5 %)

(b) $\begin{bmatrix} 3/2 & 1/2 \\ 1/2 & 3/2 \end{bmatrix}$ (5 %)

(c) and explain their physical interpretation (5%).

2. Describe the Divergence Theorem of Gauss and Stoke Theorem,

(a) from $\iiint_V (\nabla \cdot F) dV$ (10%)

(b) from $\iint (\nabla \times F) \cdot d\vec{a}$ (10%)

3. From an ordinary differential equation (ODE) of the form,

$$\frac{d^2 x}{dt^2} + \omega x = 0$$

Please describe the solutions and physical meanings with $\omega > 0$ and $\omega < 0$, respectively. (20 %)

4. Show that the wave equation,

$$\frac{\partial^2 \psi}{\partial t^2} = c^2 \left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \right),$$

has the solution of the form of $\psi = \psi_0 e^{i(k_x x + k_y y - \omega t)}$ where c is the phase

velocity of the wave (15%).

5. Find the Fourier Transform of the following equations

$$f(t) = \begin{cases} k & -a \leq t \leq a \\ 0 & t < -a \text{ \& } t > a \end{cases} \quad (15\%)$$

$$f(t) = \begin{cases} \cos kt & -a \leq t \leq a \\ 0 & t < -a \text{ \& } t > a \end{cases} \quad (15\%)$$