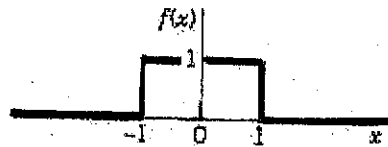※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Find the Fourier transform of

$$f(x) = \begin{cases} 1, & if \ |x| \le 1 \\ 0, & if \ |x| > 1 \end{cases}$$

<Hint> Fourier Transform has the following form:

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[\left(\int_{-\infty}^{\infty}f(v)\,e^{-i\omega v}dv\right)e^{i\omega t}\right]d\omega$$

(20%)

2. Show that the PDE

$$\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2}$$

with the boundary and initial condition of

$$u(0,t) = 0, \qquad u(L,t) = 0, \qquad for \ all \ t$$

$$u(x,0) = f(x), \qquad \frac{du(x,0)}{dt} = g(x), \qquad (0 \le x \le L)$$

has the general solution of the following form of

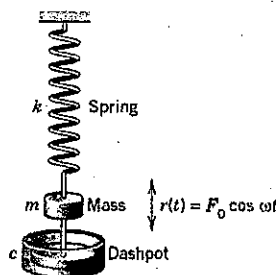$$u_n(x,t) = [A_n\cos(k_n t) + B_n cos(k_n t)]\sin(\frac{n\pi}{L}x)$$

(20%)

3. Please solve the Forced Oscillation and Resonance problem of a spring,

$$my'' + cy' + ky = F_0\cos(\omega t)$$

also describe under which circumstance resonance will occur.

(20%)

4. Solve the following ODEs

    a. $y'' + 3y' + 18y = 0$ (8%)

    b. $y'' + 16y' + 64y = 0$ (8%)

    c. $y'' + 4y = 8x$ (8%)

    d. $2\cos(x+y) - 2x\sin(x+y) - 2x\sin(x+y)y' = 0$ (8%)

    e. $x^2 y'' + 3xy' + 10y = 0$ (8%)