※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Find the Fourier series of the following function. (15%)

$$f(x) = \begin{cases} -k & if \ -2 < x < 0 \\ k & if \ \ \ 0 < x < 2 \end{cases} \quad f(x+4) = f(x)$$

2. Suppose the 1-D wave equation has the form $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$, where c is the phase velocity.
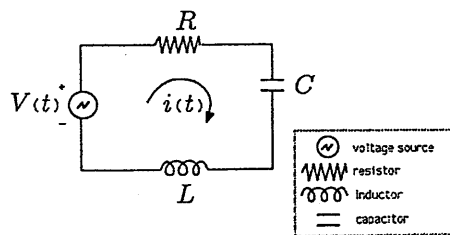
   Please show that $u(x,t)$ has the form of $u(x \pm ct)$. (15%)

3. The equation of RLC-circuit is $LI'' + RI' + \frac{1}{c}I = E'(t)$, where L is inductance 1 henrys(H),

   R is Ohm's resistance 0.05 ohms(Ω), 1/C is capacitance 25 farads(F), I(t) is electric current in the circuit, and E(t) is

   electromotive force and its derivative is the following.

$$E'(t) = \begin{cases} t + \frac{\pi}{2} & if \ -\pi < t < 0 \\ -t + \frac{\pi}{2} & if \ \ \ 0 < t < \pi \end{cases} \quad r(t+2\pi) = r(t)$$

   Find the I(t). (20%)

4. Considering vertical motions of a mass-spring system and modeled it including an external force r(t) on the right

   hand side of equation. Then we have

$$my'' + cy' + ky = r(t)$$

   r(t) is a driving force with the form of

$$r(t) = F_0 \cos \omega t$$

   Please describe in which condition resonance might occur. (15%)

5. From an ordinary differential equation (ODE) of the form,

$$\frac{d^2x}{dt^2} + \omega x = 0$$

Please describe the solutions and the corresponding physics/meanings with ω > 0 and ω < 0, respectively. (15 %)

6. Please complete the following integrals :

(1) $\int_0^{\frac{\pi}{2}} \frac{\sin 2x}{\sqrt{3-2\cos x}} dx$　　　(5%)

(2) $\int x^2 \sin x \, dx$　　　(5%)

(3) $\int \tan^{-1} x \, dx$　　　(5%)

(4) $\int x \sin x \, dx$　　　(5%)