

國立成功大學
110學年度碩士班招生考試試題

編 號： 53

系 所： 地球科學系

科 目： 應用數學

日 期： 0203

節 次： 第 4 節

備 註： 不可使用計算機

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Find the following integrals

(20% totally)

(a) $\int \ln x dx$ (5%)

(b) $\int xe^{-x} dx$ (5%)

(c) $\int \sec \theta d\theta$ (5%)

(d) $\int \frac{x^3}{\sqrt{1-x^2}} dx$ (5%)

2. Find the general solutions of following 2nd order ODEs.(20% totally)

(a). $y'' + 2y' + 3y = 0$. (5%)

(b). $y'' + y = \sec x$. (5%)

(c). $y'' + 36y = 0$. (5%)

(d). $y'' + y = \tan x$ (5%)

3. A Mass-Spring system of free oscillation(no external input) has the following equation,

$$my'' + cy' + ky = 0$$

where m is the mass of the system, c is the drag and k is the spring coefficient. Please find the general solution of the system and describe the system with under-damping, critical damping and over-damping conditions (15%).

4. Following above, If the Mass-Spring system oscillates with external force, the equation becomes

$$my'' + cy' + ky = F_0 \cos \omega t$$

where ω is the frequency of external force. Please find the general solution for the nonhomogeneous ODE (10%). Please describe under which circumstance the resonance will occur (5%).

5. Find the Fourier integral of a pulse function as follows (10%)

$$f(x) = \begin{cases} 1 & \text{if } |x| < 1 \\ 0 & \text{if } |x| > 1 \end{cases}$$

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6. Find eigen value and corresponding eigenvectors of the following matrices.

(10 %)

$$\begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

7. Show that the 2-dimensional wave equation of the following form,

$$\frac{\partial^2 \psi}{\partial t^2} = c^2 \left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \right),$$

has the general solution of the form of $\psi = \psi_0 e^{i(k_x x + k_y y - \omega t)}$ where c is the phase velocity of the wave (10%).