

國立成功大學

115學年度碩士班招生考試試題

編 號： 41

系 所： 地球科學系

科 目： 應用數學

日 期： 0204

節 次： 第 4 節

注 意： 1. 不可使用計算機
2. 請於答案卷(卡)作答，於
試題上作答，不予計分。

1. Please find the general solution of the following ordinary differential equations (15%)

(a)

$$\frac{dy}{dt} = -ky, \quad y(0) = 0.5$$

(b)

$$y' = 1 + y^2$$

(c)

$$y' = -2xy, \quad y(0) = 18$$

2. The ordinary differential equation of a spring oscillation of the following form (30%)

$$my'' + cy' + ky = 0$$

(a) Please find the general solution of the equation

(b) Under what circumstances, resonance may occur? Please also show why there is the resonance.

(c) Under what circumstances, harmonic oscillation may occur and the corresponding solution of harmonic oscillation.

3. Please derive that the general solution of the 1-D wave equation (30%)

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \quad \text{where } c \text{ is the phase velocity of the wave and the boundary/initial conditions are}$$

$$u(0, t) = 0, u(L, t) = 0 \text{ for all } t \geq 0 \quad \& \quad u(x, 0) = f(x), u'(x, 0) = g(x) \text{ for } 0 \leq x \leq L$$

$$\langle \text{Hint} \rangle \quad u(x, t) = \sum_{n=1}^{\infty} B_n \cos \frac{cn\pi t}{L} \sin \frac{n\pi x}{L}$$

4. Find the Fourier series of the following-period function ($p = 4$). (10%)

$$F(x) = 0 \text{ if } -2 < x < -1$$

$$F(x) = k \text{ if } -1 < x < 1$$

$$F(x) = 0 \text{ if } 1 < x < 2$$

5. Please find the integral of the following product of trigonometric functions (15%)

(a) $\int_{-\pi}^{\pi} \cos nx \cos mx \, dx$

(b) $\int_{-\pi}^{\pi} \cos nx \sin mx \, dx$

(c) $\int_{-\pi}^{\pi} \sin nx \sin mx \, dx$