

Total 100 points. Pocket-type scientific calculator is allowed.

### Part I: True or False

Answer T (for True) or F (for False). (2 points each question/Total 20 points.)

1. We only need to compare two sample averages to find out whether there is any difference between two groups.
2. Because normal distribution having symmetrical shape, we have  $\Pr(X < -1) = \Pr(X > 1)$ , where  $X \sim \text{Normal}(\mu, \sigma^2)$ .
3. There are about one-half of the sample values below the sample average.
4. To determine whether  $A_i$  and  $B_j$ ,  $i = 1, 2, \dots, m$ ,  $j = 1, 2, \dots, n$ , are not independent, we must to examine  $\Pr(A_i \cap B_j) = \Pr(A_i) \Pr(B_j)$  for each and all pairs of  $i$  and  $j$ .
5. The correlation coefficient tells about all the possible relationships between one variable and the other.
6. Standard deviation is not affected by small or large extreme values.
7. A hypothesis being rejected at 5% significance level implies that it will be rejected at 1% significance level.
8. The Central Limit Theroem says that the sample average would be close to the population mean as long as the sample size is large enough.
9. The *sign* of  $\beta$  (positive or negative) in the simple straight line model  $Y = \alpha + \beta x + \epsilon$  can be judged from the correlation coefficient between  $Y$  and  $x$ .
10. The width of a 95% confidence interval of a population mean, with known standard deviation, depends on the sample size.

## Part II: Calculation and Derivation

Clearly define all the notations. Show your work.  
(Points given after each question/Total 80 points)

1. The number of sample items in a production process has a Poisson distribution with a mean rate of 10 per hour. A sample is collected hourly. Production itself may be characterized as a Bernoulli process in which 20% of the items produced are defective.
  - (a) Find the unconditional probability for the number of sample items obtained is exactly 10. (5 points)
  - (b) Find the conditional probability for the number of defectives given that having 10 sample items is exactly 2. (5 points)
  - (c) Find the joint probability regarding the next hour's sample that 10 sample items are obtained *and* there are 2 defectives. (5 points)
  - (d) Find the unconditional probability that the number of defectives found in the hour is exactly 2. (10 points)
  - (e) Establish the unconditional probability distribution for the number of defectives found in the hour. (10 points)
  
2. The following data were obtained for the amount of time (seconds) taken by a proposed computer system to compile a sample of moderate-sized FORTRAN programs:  
4.7 3.0 2.3 2.9 1.0 4.4 2.4 3.5 3.9 3.9
  - (a) At the 5% significance level, test the null hypothesis that the mean compilation time for all the moderate-sized FORTRAN programs run on the system is at least 3.8 seconds. What can you conclude? (10 points)  
Clearly list your work step by step.  
Noted that  $z_{0.95} = 1.645$ ,  $t_{0.95,9} = 1.833$ ,  $t_{0.95,10} = 1.812$ .
  - (b) State any assumptions you make. (5 points)
  
3. A data clerk makes entries into a central data bank at a mean rate of 300 per hour. Suppose that 1% of all entries are in error.
  - (a) What distribution is appropriate for finding the probability of exactly 5 errors in 100 entries? Give the computation expression for finding that probability. (No need to actually compute the value.) (5 points)

- (b) What distribution is appropriate for finding the probability that the clerk will make no entries in the next minute? Give the computation expression for finding that probability. (No need to actually compute the value.) (5 points)
- (c) Determine the expected number of errors in 100 entries. (5 points)
- (d) Determine the expected number of entries in 1 minute. (5 points)
4. For the following life times (measured in 1000 hours) of 5 testing light bulbs, randomly selected from a production line,
- 1.8 0.4 1.2 3.2 0.1
- (a) What is your prediction of the life time for 90% of this kind of light bulb would fail (burn-out)? (5 points)
- (b) State the assumption(s) you can reasonably make. (5 points)