1.(10%) Use the alternating series test to show that the improper integral

$$\int_0^\infty \frac{\sin x}{x}\,dx$$

converges.

2. Find the following indefinite integrals.

(a)(5%) $\int \sqrt{\frac{1+x}{1-x}}\,dx$.

(b)(5%) $\int \frac{x^5}{(1+x^2)^4}\,dx$.

3.(15%) Show that $\sum_{n=1}^{\infty} \frac{1}{n(n+1)\cdots(n+k)} = \frac{1}{k\cdot k!}$ for any positive integer k.

4. Evaluate the following limits.

(a)(5%) $\lim_{x\to 0^+} \frac{1}{x}\int_0^x (1+\sin 2t)^{1/t}\,dt$.

(b)(5%) $\lim_{x\to 1} \ln x \tan(\pi x/2)$.

5.(10%) Find the absolute maximum and minimum of $f(x) = \tan^{-1}(\frac{1-x}{1+x})$ on the interval $[0,1]$.

6.(10%) Find the maximum and minimum directional derivatives of the function defined by $f(x) = x^2 + y^2 + x - y + 1$ at the point $(1,-1)$.

7.(10%) Compute the multiple integral $\int_0^4 \int_{y/2}^2 e^{x^2}\,dx$.

8.(10%) Find the indefinite integral $\int \frac{\sqrt{4-x^2}}{x^2}\,dx$.

9.(10%) Show that $\sum_{n=3}^{\infty} \frac{1}{(\ln n)^{\ln\ln n}}$ diverges. Hint: show that $(\ln\ln n)^2 \le \ln n$ for large n.

10.(5%) Let $f(x,y) = \frac{x+y}{x-y}$ and $\vec{v} = \frac{1}{2}\vec{i} + \frac{\sqrt{3}}{2}\vec{j}$. Find the directional derivative of f at the point $(1,-1)$ in the direction of $\vec{v}$.

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