

1. The following data is listed as

x	1	2	3	4
y	3	5	6	2

Questions:

- (a) Set up the polynomial such that $y(x_i) = y_i$. (11%)
(b) Further, what are the values of a and b in $y = ax + b$ to least square approximate the data listed in the table. (8%)

2. A surface is defined by $z = x^2 + y^2$. Calculate the surface area defined on the domain $0 \leq x^2 + y^2 \leq b^2$. (20%)

3. The differentiation equation is given as $y'' + 3y' + 2y = e^x$.

Questions:

- (a) What is the homogeneous solution? (5%)
(b) What is the particular solution? (5%)
(c) If the initial conditions are $y(0) = 1$ and $y'(0) = 1$, then what is the complete solution? (8%)

4. Solve the system of equations

$$\begin{aligned}8x_1 - 4x_2 + 3x_3 &= 0 \\x_1 + 5x_2 - x_3 &= -5 \\-2x_1 + 6x_2 + x_3 &= -4.\end{aligned}\quad (20\%)$$

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(背面仍有題目,請繼續作答)

5. Suppose $U_1(x, y)$ to be the solution of the partial differential equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad 0 \leq x, y \leq a,$$

with the boundary conditions

$$u(x, 0) = g(x), \quad u(x, a) = 0, \quad u(0, y) = 0, \quad u(a, y) = 0.$$

Questions:

(a) What is the solution $U_2(x, y)$ in terms of $U_1(x, y)$ for these boundary conditions

$$u(x, 0) = 0, \quad u(x, a) = g(x), \quad u(0, y) = 0, \quad u(a, y) = 0? \quad (8\%)$$

(b) What is the solution $U_3(x, y)$ in terms of $U_1(x, y)$ for these boundary conditions

$$u(x, 0) = 0, \quad u(x, a) = 0, \quad u(0, y) = g(y), \quad u(a, y) = 0? \quad (5\%)$$

(c) What is the solution $U_4(x, y)$ in terms of $U_1(x, y)$ for these boundary conditions

$$u(x, 0) = 0, \quad u(x, a) = 0, \quad u(0, y) = 0, \quad u(a, y) = g(y)? \quad (5\%)$$

(d) What is the solution $U_5(x, y)$ in terms of $U_1(x, y)$ for these boundary conditions

$$u(x, 0) = g(x), \quad u(x, a) = 3g(x), \quad u(0, y) = 2g(y), \quad u(a, y) = 4g(y)? \quad (5\%)$$