

系所組別 地球科學系甲、乙組

考試科目 · 微積分

考試日期 · 0306 · 節次 · 4

※ 考生請注意：本試題 可 不可 使用計算機

1. (10) Show that for any real number
- $a$
- there exists a real number
- $c$
- such that

$$c^{2009} + 2c^{2008} + \cdots + 2008c^2 + 2009c = a$$

2. (10) Evaluate

$$\int_0^{\pi/8} (\tan^2 2x + \sec^2 2x) dx$$

3. (10) Find the area of the region between the inner and outer loops of
- $r = 1 - \cos \theta$

4. (10)

$$f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

- (a) (5) Find
- $f'(0)$

- (b) (5) Does
- $f$
- attain its local extreme value at
- $x = 0$
- ? Justify your answer!

5. (10) Find the limit

$$\lim_{m \rightarrow \infty} \left[ \lim_{n \rightarrow \infty} \cos^{1/n}(m! \sqrt{11} \pi) \right]$$

6. (10) Determine whether the integral converges. If it converges, give the value of the integral.

- (a) (5)

$$\int_{-\infty}^{\infty} \frac{r}{r^2 + x^2} dx, \quad r > 0$$

- (b) (5)

$$\int_{-\infty}^{\infty} \frac{2x}{1 + x^2} dx$$

7. (10) Find the Taylor series expansion of
- $f(x) = \sin 2x$
- in powers of
- $(x - \pi/4)$
- and give the radius of convergence.

8. (10) Evaluate

$$\int_{-1}^1 \int_0^{\sqrt{1-x^2}} \arctan\left(\frac{y}{x}\right) dy dx$$

9. (10) Let
- $x_1, x_2, \dots, x_n$
- be nonnegative numbers such that
- $x_1 + x_2 + \cdots + x_n = k, k$
- a constant. Prove that

$$(x_1 x_2 \cdots x_n)^{1/n} \leq \frac{1}{n} (x_1 + x_2 + \cdots + x_n)$$

10. (10) Determine whether
- $f$
- is continuous at
- $(0, 0)$
- where

- (a) (5)

$$f(x) = \frac{2x^2 y}{x^4 + y^2}$$

- (b) (5)

$$f(x) = \begin{cases} \frac{x^2 y^2}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

Justify your answers!