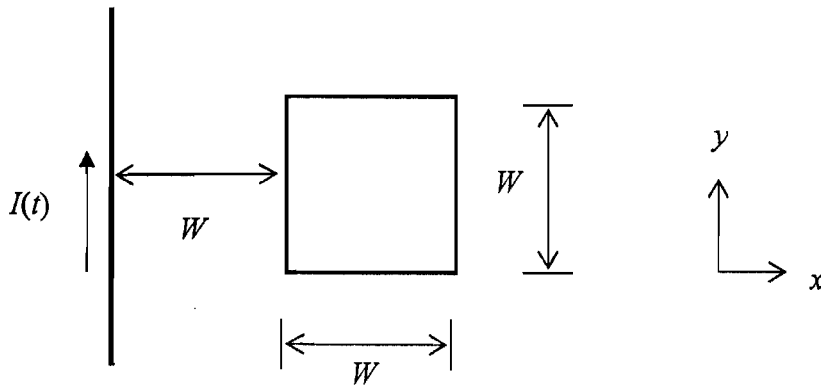


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**Part I: 選擇題 (每題五分)**

- Assuming that the electric field intensity  $\vec{E} = 10x \hat{i} + 10y \hat{j} + 10 \hat{k}$  (V/m), find the total electric charge contained inside a cubical volume of 20 cm on a side centered symmetrically at the origin.  
 (a) 20 C (b) 0.16 C (c)  $1.42 \times 10^{-12}$  C (d)  $1.77 \times 10^{-10}$  C (e)  $8.86 \times 10^{-11}$  C
- A very long, fixed straight wire with a time-dependent current  $I$  points to  $+y$  direction. The current  $i$  varies with time as  $I(t) = 3t^2 + 2$  Amp. A square loop with side  $W$  located at the right hand side of the straight wire, as shown below. If the square loop has the side  $W = 1$  m and a total resistance  $R = 1.2 \Omega$ , What is the induced current on the loop at  $t = 1$  sec?  
 (a)  $6.9 \times 10^{-7}$  A counterclockwise (b)  $8.3 \times 10^{-7}$  A counterclockwise (c)  $2 \times 10^{-7}$  A counterclockwise  
 (d)  $6.9 \times 10^{-7}$  A clockwise (e)  $8.3 \times 10^{-7}$  A clockwise (f)  $2 \times 10^{-7}$  A clockwise



- Same as the previous case. What is the magnetic dipole moment  $\vec{m}$  of the loop (magnitude and direction) in units of Amp-m<sup>2</sup> at  $t = 2$  sec?  
 (a)  $6.9 \times 10^{-7} \hat{z}$  (b)  $1.4 \times 10^{-6} \hat{z}$  (c)  $2 \times 10^{-6} \hat{z}$  (d)  $6.9 \times 10^{-7} -\hat{z}$  (e)  $1.4 \times 10^{-6} -\hat{z}$
- The  $x$ -polarized uniform plane wave with frequency 150 MHz propagates in air along the  $z$ -direction and impinges on a perfectly conducting plane at  $z = 0$ . Assuming the amplitude of the  $\vec{E}$  field for the incident wave is 12 (mV/m), what is the expression for the reflected  $\vec{H}(t, z)$  field in A/m?  
 (a)  $(10^{-4}/\pi)\cos(1.5 \times 10^7 t - \pi z)\hat{y}$  (b)  $(10^{-4}/\pi)\cos(1.5 \times 10^7 t + \pi z)\hat{y}$   
 (c)  $(10^{-4}/\pi)\cos(3\pi \times 10^8 t + 2\pi z)\hat{y}$  (d)  $(10^{-4}/\pi)\cos(3\pi \times 10^8 t - \pi z)\hat{y}$  (e)  $(10^{-4}/\pi)\cos(3\pi \times 10^8 t + \pi z)\hat{y}$

(背面仍有題目,請繼續作答)

系所組別： 物理學系

考試科目： 電磁學

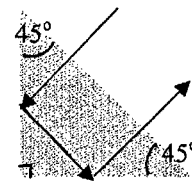
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5. For the following two-dipole systems, which configuration has the lowest electrostatic energy?  
In each case, two identical electric dipoles separate with the same distance  $R$  from each center point with  $R \gg d$ , where  $d$  is the length of the electric dipole.

(a)  $\rightarrow \leftarrow$  (b)  $\rightarrow \rightarrow$  (c)  $\uparrow \uparrow$  (d)  $\uparrow \downarrow$  (e)  $\rightarrow \uparrow$

6. A time-dependent voltage source  $V(t) = \pi t^3 + 2$  Volt is connected across a parallel-plate capacitor with separation  $d = 3$  mm and surface area  $S = 1$  m<sup>2</sup>. What is the value of the displacement current between the plates at  $t = 6$  sec? (a)  $10^{-4}$  A (b)  $10^{-5}$  A (c)  $10^{-6}$  A (d)  $10^{-7}$  A (e)  $10^{-8}$  A.
7. A triangular prism shown below has a relative dielectric constant  $\epsilon_r = 4$  and  $\mu_r = 1$ . What is the critical angle  $\theta_c$  for the incident light from the prism to air?  
(a)  $30^\circ$  (b)  $37^\circ$  (c)  $45^\circ$  (d)  $53^\circ$  (e)  $60^\circ$



8. Same as the previous case. What is the percentage of the incident light power reflected back by the prism? The transmission coefficient for a normal incident light is  $\tau = 2\eta_2/(\eta_2 + \eta_1)$ .  
(a) 0.25 (b) 0.50 (c) 0.67 (d) 0.79 (e) 0.89
9. In an air-filled rectangular cavity resonator has dimensions  $a = b = 1.5$  cm and  $d = 3$  cm. The  $z$ -component  $H$ -field for the TE modes in a Cartesian coordinate is

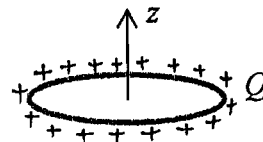
$H_z(x, y, z) = H_0 \cos\left(\frac{m\pi}{a}x\right) \cos\left(\frac{n\pi}{b}y\right) \sin\left(\frac{p\pi}{d}z\right)$ , where  $m$ ,  $n$ , and  $p$  are integrals. The resonant frequency is given as  $f_{mnp} = \frac{1}{2\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{d}\right)^2}$ . What is the lowest resonant frequency of the TE modes? (a) 5 GHz (b) 10 GHz (c)  $5\sqrt{5}$  GHz (d)  $10\sqrt{2}$  GHz (e) 15 GHz

10. Assuming that the total radiated power from an oscillating electric dipole system is 0.1 W. What is the new radiated power when the oscillating angular frequency of the dipole increases from  $\omega$  to  $2\omega$ ? (a) 0.2 W (b) 0.4 W (c) 0.8 W (d) 1.6 W (e) 3.2 W

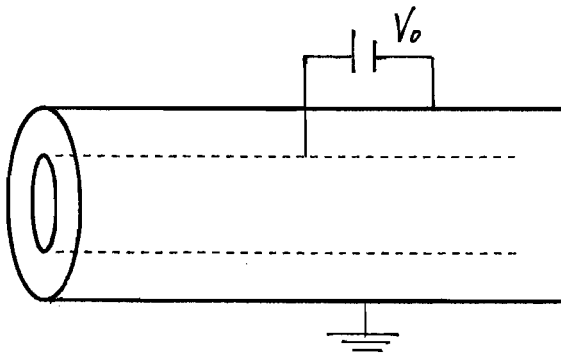
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**Part II: 計算題**

11. (15 Points) A total positive charge  $Q$  is uniformly distributed on a ring of radius  $R$ . (a) Calculate the electric potential along the  $z$ -axis as a function of  $z$ , and the maximum electric potential of this charged ring. (b) Compute the  $z$ -component electric field  $E_z$  as a function of  $z$ . (c) What is the value of the maximum  $E_z$  of this charged ring? (d) An electron with charge  $e$  and mass  $m$  is released from the point  $z \ll R$  along the axis of the ring. The motion of the electron can be described as a simple harmonic motion (SHM). Derive the equation of motion for this SHM. (e) What is the angular frequency  $\omega$  for this SHM? Express your answers in terms of  $\epsilon_0$ ,  $Q$ ,  $e$ ,  $m$ , and  $R$ .



12. (10 Points) Assuming that the space between the inner and outer conductors of a long coaxial cylindrical structure is filled with an electron cloud having a volume density charge  $\rho(r) = A/r$  for  $a < r < 2a$ , where  $a$  and  $2a$  are the radii of the inner and outer conductors respectively. The inner conductor is maintained at a constant potential  $V_0$  while the outer conductor is grounded. Determine the potential distribution in the region  $a < r < 2a$  by solving the Poisson's equation with numerical values of  $a = 2$  m,  $A = \epsilon_0 \ln 2$  C/m<sup>2</sup>, and  $V_0 = \ln 2$  Volt. ( $\epsilon_0 = 1/(36\pi \times 10^9)$  F/m)



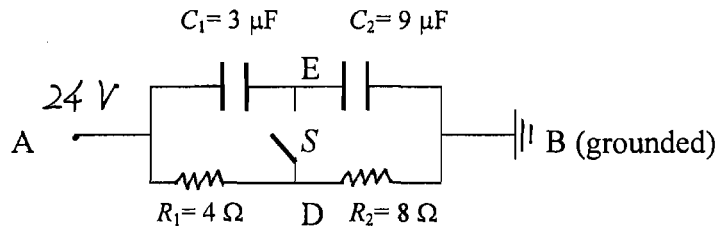
In cylindrical coordinates,

$$\nabla^2(r, \phi, z) = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \phi^2} + \frac{\partial}{\partial z^2}$$

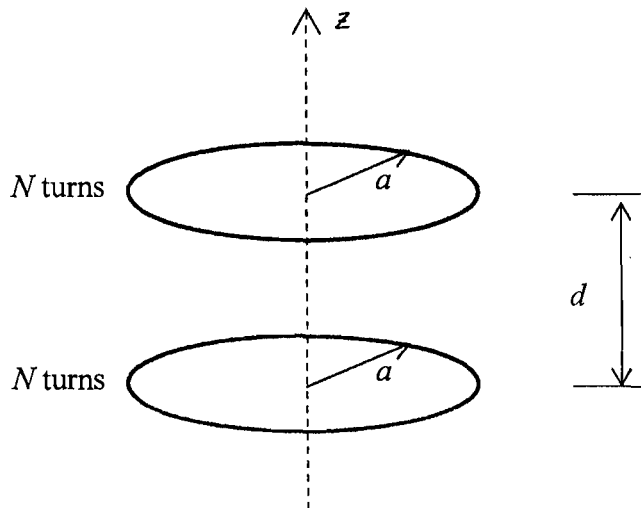
(背面仍有題目, 請繼續作答)

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13. (10 Points) The capacitors shown in the accompanying figure are all uncharged when 24 V is applied between points A and B with the switch open. (a) Find the potential difference between points D and E,  $V_D - V_E$  while the switch  $S$  remains opened. (b) If the switch  $S$  is closed at  $t=0$ , what is the final electric potential energy stored in  $C_1$  as  $t \rightarrow \infty$  ?



14. (15 Points) Two identical coaxial coils, each of  $N$  turns and radius  $a$ , are separated by a distance  $d$ , as shown below. A current  $I$  flows in each coil in the same direction. (a) Express the magnitude of the magnetic field  $B_z$  at the  $z$  axis between the coils. (b) Show that  $dB_z/dz$  vanishes at the midpoint  $z = d/2$ . (c) Find the relation between  $a$  and  $d$  such that  $d^2B_z/dz^2$  also vanishes at the midpoint. Hint: For a single coil of radius  $a$ , with a current  $I$ ,  $B_z(z) = \frac{\mu_0 I}{2} \frac{a^2}{(a^2 + z^2)^{3/2}}$ .



\*Useful values:

$$\epsilon_0 = 1/(36\pi \times 10^9) \text{ F/m}; \mu_0 = 4\pi \times 10^{-7} \text{ H/m}$$