

系所組別：物理學系

考試科目：電磁學

考試日期：0226，節次：2

1. [20 %] An uncharged metal sphere of radius R is placed in an otherwise uniform electric field $\vec{E} = E_0\vec{k}$. The field will push positive charge to the "northern" surface of the sphere, leaving a negative charge on the "southern" surface.
 - (a) Find the potential in the region outside the sphere. (10 %)
 - (b) Find the induced charge density on the surface. (10 %)
2. [30 %] A spherical shell of radius R , carrying a uniform surface charge σ , is set spinning at angular velocity ω .
 - (a) Find the vector potential it produces inside and outside the rotating shell. (10 %)
 - (b) What is the magnetic dipole moment of the rotating shell? (10 %)
 - (c) Find the electric field inside and outside the sphere if the angular velocity $\omega(t)$ changes slowly with time. (10 %)
3. [10 %] (a) State the Biot-Savart law which gives the magnetic field \vec{B} at a distance r from a current element. Hence obtain an expression for the magnetic field \vec{B}_Q due to a point charge Q moving with constant velocity \vec{v} (assuming non-relativistic). (5 %)
 - (b) Point charges Q and Q' are constrained to move along the x - and y -axes, respectively, with the same uniform speed v . At $t = 0$ both charges are at the origin. At time t calculate the Lorentz force \vec{F} on Q' due to the magnetic field of Q . (5 %)
4. [10 %] A long coaxial cable carries current I (the current flows down the surface of the inner cylinder, radius a , and back along the outside cylinder, radius b) as shown in Fig. 1. Find the energy stored in a section of length l .

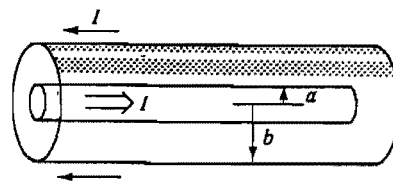


Fig. 1

5. [20 %] Find the charge and current distributions that would give rise to the potentials $V = 0$,

$$\vec{A} = \begin{cases} \frac{\mu_0 \alpha}{4c} (ct - |x|)^2 \vec{k}, & |x| < ct \\ 0, & |x| > ct \end{cases}$$

(here α is some constant, and c is shorthand for $1/\sqrt{\epsilon_0\mu_0}$).

6. [10 %] Find the Lienard-Wiechert potential of a point charge moving with constant velocity \vec{v} .