

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (a) Write down the Lorentz transformation matrix for the boost in the x-direction. (10%)  
 (b) If  $E$  is the energy of a relativistic particle, how large is its de Broglie wavelength? (10%)
  
2. Consider a particle with energy  $E$  is incident from the left-side. The particle is influenced by the following potential:  $V(x) = V_0[\delta(x) + \delta(x - a)]$ .  
 (a) Write down the wave-functions with undetermined coefficients in different regions ( $x < 0$ ,  $0 < x < a$ ,  $x > a$ ). (10%)  
 (b) Find out the transmission coefficient. (10%)
  
3. (a) Calculate the Expectation values for kinetic energy  $K$  and potential energy  $V$  in the ground state of the hydrogen energy. (10%)  
 (b) Calculate  $\langle z^2 \rangle$  for an electron in the ground state of hydrogen. (10%)
  
4. Consider a particle in a potential
 
$$V(x) = \begin{cases} \frac{1}{2}kx^2, & \text{for } x < 0 \\ \infty, & \text{for } x > 0 \end{cases}$$
 (a) What is the energy of the first-excited state? (5%)  
 (b) What is the expectation value  $\langle x^2 \rangle$  for the first-excited state? (10%)
  
5. Imagine two non-interacting particles, each of mass  $m$ , in the infinite square well with width  $a$ . If one is in the ground state  $\psi_1$  and the other is in the first excited state  $\psi_2$ , calculate  $\langle x_1 x_2 \rangle$  assuming they are identical fermions. (10%)
  
6. (a) Write down the spin angular momentum operators ( $S_x$ ,  $S_y$ , and  $S_z$ ) of a spin-1/2 particle. (5%)  
 (b) Suppose a spin-1/2 particle is in the state  $\frac{1}{\sqrt{6}} \begin{pmatrix} 1-i \\ 2 \end{pmatrix}$ . What are the probabilities of getting  $-\hbar/2$  if you measure  $S_x$ ? (10%)