編號： 40

國立成功大學 105 學年度碩士班招生考試試題

系　　所：物理學系

考試科目：物理數學

考試日期：0228，節次：1

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. ϵ_{ijk} and δ_{lm} are the Levi-Civita symbol and Kronecker delta in 3-dimensional space, respectively; using the Kronecker delta to express following results:
   (a) $\sum_k \epsilon_{ijk}\epsilon_{lmk}$,
   (b) $\sum_{jk} \epsilon_{ijk}\epsilon_{ljk}$. (10 points each)

2. (a) A and B are two matrices, under the similarity transformation $A' = UAU^{-1}$ and $B' = UBU^{-1}$, show that $trace\ (AB)$ is an invariant quantity. (5 points)
   (b) H is a nonsingular Hermitian matrix, show that $\det(\exp(H)) = \exp\ (trace(H))$. (10 points)

3. ∇ is the gradient operator and V is a vector field, show that
$$\mathbf{V}\times(\nabla\times\mathbf{V}) = \frac{1}{2}\nabla(\mathbf{V}^2) - (\mathbf{V}\cdot\nabla)\mathbf{V}.$$
   (10 points)

4. Find the general solution for the ordinary differential equation, $y'' - 2\,y' + y = 0$. (10 points)

5. $f(z)$ is a function in complex plane.
   (a) Explain the meaning of an analytic function. (5 points).
   (b) What condition can lead $f(z)$ to be an analytic function? (15 points)

6. (a) Explain the residue theorem. (10 points)
   (b) Applying the residue theorem in complex variable theory, calculate the integral
$$\int_0^\infty \frac{\sin x}{x}\,dx\,.$$
   (15 points)