編號： 42

# 國立成功大學 105 學年度碩士班招生考試試題

系　所：物理學系

考試科目：近代物理學　　　　考試日期：0228，節次：3



※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Show that the propagation of electromagnetic waves in Maxwell's equations is Lorentz invariant. (20%)

2. If a system has non-degenerate energy levels with energy $E=\left(n+\frac{1}{2}\right)\hbar\omega$,

$n=0, 1, 2, 3, \dots\dots\dots, \infty$, $\hbar\omega/k=1$ ($k$ is the Boltzmann's constant = $1.38*10^{-23}$ J/K),

calculate the probability that the system is in the $n$ = 10 state when the temperature

(a) $T=300$ K, (10%)

(b) $T=0$ K, (5%)

(c) $T=\infty$. (5%)

3. A system has quantum mechanical Hamiltonian $H=-\frac{\hbar^2}{2m}\nabla^2+r^2\left(1-\frac{5}{6}\sin^2\theta\sin^2\phi\right)$.

Calculate the two lowest lying energy eigenvalues of the system. (20%)

4. Derive an expression for the Bohr magneton. (20%)

5. If the wave function of a particle in a spherically symmetric potential is:

$$\psi(x,y,z)=(xy+yz+zx)e^{-r^2}$$

, show the probability of angular momentum for $l=0$ and $l=1$ is zero, and for $l=2$ is unity. (20%)

hint: the spherical harmonics:

$$Y_0^0(\theta,\varphi)=\frac{1}{2}\sqrt{\frac{1}{\pi}}$$

$$Y_1^{-1}(\theta,\varphi)=\frac{1}{2}\sqrt{\frac{3}{2\pi}}(\sin\theta)e^{-i\varphi}$$

$$Y_1^{0}(\theta,\varphi)=\frac{1}{2}\sqrt{\frac{3}{\pi}}(\cos\theta)$$

$$Y_1^{-1}(\theta,\varphi)=\frac{-1}{2}\sqrt{\frac{3}{2\pi}}(\sin\theta)e^{i\varphi}$$

$$Y_2^{-2}(\theta,\varphi)=\frac{1}{4}\sqrt{\frac{15}{2\pi}}(\sin^2\theta)e^{-2i\varphi}$$

$$Y_2^{-1}(\theta,\varphi)=\frac{1}{2}\sqrt{\frac{15}{2\pi}}(\sin\theta\cos\theta)e^{-i\varphi}$$

$$Y_2^{0}(\theta,\varphi)=\frac{1}{4}\sqrt{\frac{5}{\pi}}(3\cos^2\theta-1)$$

$$Y_2^{1}(\theta,\varphi)=\frac{-1}{2}\sqrt{\frac{15}{2\pi}}(\sin\theta\cos\theta)e^{i\varphi}$$

$$Y_2^{2}(\theta,\varphi)=\frac{1}{4}\sqrt{\frac{15}{2\pi}}(\sin^2\theta)e^{2i\varphi}$$