※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Find the value of the integral $\int_S (\nabla \times \mathbf{A}) \cdot d\mathbf{a}$ if the vector $\mathbf{A} = y\mathbf{i} + z\mathbf{j} + x\mathbf{k}$ and S is the surface defined by the paraboloid $z = 1 - x^2 - y^2$, where $z \geq 0$. (12 %)

2. Prove the following statements:

   (a) The eigenvalues of a Hermitian matrix are real. (6 %)

   (b) The eigenvectors of a Hermitian matrix corresponding to different eigenvalues are orthogonal. (6 %)

   (c) If A and B are Hermitian matrices, then AB is not Hermitian unless A and B commute. (6 %)

3. Find the eigenvalues λ_1, λ_2, and the corresponding eigenvectors of the two-dimensional rotation matrix

$$C = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}. \quad (8\%)$$

Find the unitary matrix U, such that

$$U^+ C U = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}. \quad (2\%)$$

4. Consider the differential equation $\frac{d^2 y}{dx^2} + 8\frac{dy}{dx} + (\lambda + 16)y = 0$ as an eigenvalue equation defined on the range $0 \leq x \leq \pi$ with boundary condition $y(0) = y(\pi) = 0$. Find the eigenvalues and eigenfunctions. (15 %)

5. Evaluate the following integrals:

   (a) $\int_0^\infty \phi(x)\delta(x^2 - a^2)dx$　(10%),　　(b) $\oint_{|z|=1} \frac{\cos z}{z^3}dz$　(10%).

6. Let $f(z) = u(x,y) + iv(x,y)$ be an analytic function. If $u(x,y) = xy$, find $v(x,y)$ and $f(z)$. (10 %)

7. Consider a resistance R and an inductance L connected in series with a voltage $V(t)$. The equation governs the current is

$$L\frac{dI}{dt} + RI = V(t).$$

Suppose $I(0) = 0$ and $V(t)$ is a voltage impulse at $t = t_0$ given by $V(t) = A\delta(t - t_0)$. Find the current by the Laplace transform method. (15 %)