國立成功大學 106 學年度碩士班招生考試試題

系　所：物理學系

考試科目：近代物理學

考試日期：0214，節次：3

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. According to the energy-time uncertain principle, resolving the energy difference before and after the emergence of virtual particles requires waiting for a certain amount of time. When using this principle in a reverse way (i.e., to NOT detect a violation of energy conservation), one can determine the lifetime of particles. How does this lifetime depend upon the particles' mass and momentum (or wavelength)? (10%)

2. (a) Derive Planck's expression for the average energy $\bar{\mathcal{E}} = \frac{h\nu}{Exp\left(\frac{h\nu}{kT}\right)-1}$ using Planck's postulate and a special form of Boltzmann distribution $p(\varepsilon) = \frac{Exp\left(\frac{-\varepsilon}{kT}\right)}{kT}$. (10%) (b) Obtain the Planck blackbody spectrum by multiplying the result of (a) by $N(\nu)d\nu$. Here $N(\nu)d\nu$ is the number of allowed frequencies between ν and $\nu + d\nu$, which you must compute. (10%)

3. The Wilson-Sommerfeld rule can be stated as follow: For any physical system in which the coordinates are periodic functions of time, there exists a quantization condition for each coordinate. Theses quantization conditions take the form, $\oint p_q dq = n_q h$, where q is the periodic coordinate, p_q is the momentum associated with that coordinate, and n_q is a quantum number which takes on discrete values. Show that the Bohr and Planck quantization postulates can be derived from this rule. (20%)

4. A particle is in an infinite square well potential with walls at $x = 0$ and $x = L$. If the particle is in the state $\psi(x) = A\sin(\frac{3\pi x}{L})$, where A is a constant, what is the probability that the particle is between $x = L/3$ and $x = 2L/3$? (10%)

5. Suppose the state of a spin $\frac{1}{2}$ particle is described by the spinor $\chi = A\begin{bmatrix} 1+i \\ 2 \end{bmatrix}$, where A is a normalization constant. Compute the probability of finding the particle with z-component spin $S_z = -\frac{1}{2}\hbar$. (10%)

6. (a) What are selection rules for the allowed transitions of one-electron atoms? (5%) (b)What are physical principles behind these rules? (10%) Hint: One of clues which can be considered is the photon emission rate or, equivalently, the atomic transition rate, $R = \frac{4\pi^3\nu^3}{3\epsilon_0 hc^3}p^2$, where p is the matrix element of the electric dipole moment taken between initial and final states.

7. What is the fine-structure constant and what is the physics behind the atomic spectral phenomena it was introduced to explain? (5%)

8. An electron with total energy E in the region $x < 0$ is moving in the $+\vec{x}$ direction. It encounters a step potential at $x = 0$. The wave function for $x \le 0$ is given by

   $\psi = \mathrm{AExp}(ik_1x) + \mathrm{Exp}(-ik_1x)$, where $k_1 = \sqrt{2mE}/\hbar$; and the wave function for $x > 0$ is given by

   $\psi = \mathrm{CExp}(ik_2x)$, where $k_2 = \sqrt{2m(E - V_0)}/\hbar$. Find the reflection coefficient for the system. (10%)