國立成功大學 107 學年度碩士班招生考試試題

系　所：物理學系

考試科目：物理數學

考試日期：0206，節次：1

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. A vector **B** is formed by $\boldsymbol{B} = \boldsymbol{\nabla} u \times \boldsymbol{\nabla} v$, where $u\ and\ v$ are scalar functions. Show that:
   (a) **B** is solenoidal; (10 points)
   (b) $\boldsymbol{B} = \boldsymbol{\nabla} \times \mathbf{A}$, where the vector $\boldsymbol{A}$ can be expressed as:

$$A = \frac{1}{2}(u\boldsymbol{\nabla} v - v\boldsymbol{\nabla} u). \text{ (10 points)}$$

2. Find the Fourier transform of the function $f(x) = \frac{1}{\sqrt{2\pi}} \frac{2\alpha}{\alpha^2+x^2}$ with $\alpha > 0$. (15 points)

3. The beta function is defined as: $B(p,q) = \Gamma(p)\Gamma(q)/\Gamma(p+q)$. The integral representation of gamma function can be expressed as: $\Gamma(z) = \int_0^\infty e^{-t} t^{z-1} dt$, $Re(z) > 0$. Show that:

(a) $B(p+1, q+1) = 2\int_0^1 x^{2p+1}(1-x^2)^q\, dx$; (15 points)

(b) $\int_{-1}^{1}(1-x^2)^n dx = \frac{2\,(2n)!!}{(2n+1)!!}$. (15 points)

4. The differential equation $P(x,y)dx + Q(x,y)dy = 0$ is exact, where its solution $\phi(x,y)$ can match $d\phi(x,y) = 0$. Show that the solution can be expressed as:

$$\phi(x,y) = \int_{x_0}^{x} P(x',y)dx' + \int_{y_0}^{y} Q(x_0,y')dy' = constant. \text{ (20 points)}$$

5. (a) **A** is a non-Hermitian operator. Verify that $\boldsymbol{A} + \boldsymbol{A}^\dagger$ and $i(\boldsymbol{A} - \boldsymbol{A}^\dagger)$ are Hermitian operators. (8 points)
   (b) Using the result in (a), show that any non-Hermitian operator could be expressed as the combination of the two Hermitian operators. (7 points)