國立成功大學

110學年度碩士班招生考試試題

編　號：37

系　所：物理學系

科　目：物理數學

日　期：0203

節　次：第 1 節

備　註：不可使用計算機

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Working entirely in circular cylindrical coordinates, (15%)

   (a) show that the position vector $\vec{r} = \hat{e}_\rho \rho + \hat{e}_z z$. (5%)

   (b) calculate $\nabla \cdot \vec{r}$ and $\nabla \times \vec{r}$. (10%)

2. Consider the differential equation $x\dfrac{dy}{dx} - xy = y$. (10%)

   (a) Find the solution $y(x)$ containing one arbitrary constant C. (5%)

   (b) Find the value of C if the following boundary condition is satisfied: $y = 1$ when $x = 1$. (5%)

3. Consider the matrix $A = \begin{pmatrix} 3 & 1-i \\ 1+i & 2 \end{pmatrix}$. (25 points)

   (a) Verify if A is Hermitian. (5%)

   (b) Find the eigenvalues of A and the corresponding normalized eigenvectors. Check if the eigenvectors are orthogonal. (15%)

   (c) Write a unitary matrix U such that $U^{-1}AU$ is diagonal. (5%)

4. Evaluate $\oint_C \dfrac{dz}{z^2 - 1}$, where C is the circle $|z-1| = 1$ in the complex plane. (10%)

5. Consider the function $f(x) = \begin{cases} 0, & -\pi < x < 0 \\ \sin x, & 0 < x < \pi \end{cases}$. (20%)

   (a) Find the Fourier series representation of $f(x)$. (15%)

   (b) Using the result of (a), calculate $\displaystyle\sum_{n=1}^{\infty} \dfrac{1}{(2n)^2 - 1}$. (5%)

6. Consider $f(r) = 1/r$ and the Laplace's equation $\nabla^2 \Psi = 0$. (10%)

   (a) Verify that $f(r)$ with $r \neq 0$ is a solution of Laplace's equation. (5%)

   (b) Find the expression of $\nabla^2 f(r)$, including $r = 0$. (5%)

7. $\Phi(x,h) = \left(1 - 2xh + h^2\right)^{-1/2}$ with $|h| < 1$ is the generating function for Legendre polynomials such that

   $\Phi(x,h) = P_0(x) + hP_1(x) + h^2 P_2(x) + \cdots = \displaystyle\sum_{m=0}^{\infty} h^m P_m(x)$. Calculate $P_{2n+1}(0)$ and $P_{2n}(0)$. (10%)