國立成功大學 110 學年度碩士班招生考試試題

系　所：物理學系

考試科目：近代物理學

考試日期：0203，節次：3

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

Useful constants:

$c = 3 \times 10^{8} m/s;\quad \hbar = 1.05 \times 10^{-34} J \cdot s; 1J = 6.24 \times 10^{-18}\, eV; 1nm = 10^{-9} m.$

1. Use the uncertainty principle to make an order of magnitude estimate for the kinetic energy (in eV) of an electron in a hydrogen atom. (10 points)

2. (a) By using the de Broglie relation, derive the Bohr condition $mvr = n\hbar$ for the angular momentum of an electron in a hydrogen atom. (10 points)

   (b) Use this expression to show that the allowed electron energy states in hydrogen atom can be written as $E_n = -\frac{me^4}{8\epsilon_0^2 h^2 n^2}$ (15 points)

3. A non-relativistic particle of mass m is held in a circular orbit around the origin by an attractive force $f(r) = -kr$, where k is a positive constant:

   (a) Show that the potential energy can be written $U(r) = k\frac{r^2}{2}$, where U(r)=0 when r=0. (5 points)

   (b) Assuming the Bohr quantization of the angular momentum of the particle, show that the radius r of the orbit of the particle and speed v of the particle can be written as:

   $$v^2 = \left(\frac{n\hbar}{m}\right)\left(\frac{k}{m}\right)^{\frac{1}{2}}, \qquad r^2 = \left(\frac{n\hbar}{k}\right)\left(\frac{k}{m}\right)^{\frac{1}{2}},$$

   where n is an integer. (10 points)

   (c) Using (b) results, calculate the total energy of the particle. (10 points)

   (d) If $m = 3 \times 10^{-26} kg$ and $k = 1180\ Nm^{-1}$, determine the wavelength of the photon in nm, which will cause a transition between successive energy levels. (5 points)

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4. Find the flux of particles represented by the wave function
$$\Psi = A\,e^{ikx} + Be^{-ikx} \quad \text{(10 points)}$$

5. Show that a free electron cannot absorb a photon and conserve both energy and momentum in the process. Hence, the photoelectric process requires a bound electron. (10 points)

6. Show that the de Broglie wavelength of a particle, of charge *e*, rest mass m_0, moving at relativistic speed is given as a function of the accelerating potential *V* as:
$$\lambda = \frac{h}{\sqrt{2m_0 eV}}\left(1 + \frac{eV}{2m_0 c^2}\right)^{-\frac{1}{2}} \quad \text{(15 points)}$$