

國立成功大學

113學年度碩士班招生考試試題

編 號：37

系 所：物理學系

科 目：物理數學

日 期：0202

節 次：第 1 節

備 註：不可使用計算機

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Estimate gradient ∇ of the function $\varphi(x, y, z)$ [10 points]

$$\varphi(x, y, z) = \frac{z x^2}{x^2 + y^2 + z^2}$$

2. Solve $y(x)$ for 1st-order ordinary differential equation (ODE)

$$(1 + y^2)dx + xydy = 0 \text{ with } y(3) = 0 \text{ [10 points]}$$

3. Solve both complementary function $y_c(x)$ and particular function $y_p(x)$ of following 2nd-linear ODEs :

$$(a) y'' + 3y' + 2y = e^{3x} \text{ [10 points]}, (b) y'' + 3y' + 2y = 1 + x \text{ [10 points]}$$

4. (a) Find eigenvalue and eigenvectors of the matrix $\begin{bmatrix} 1 & 3 & 2+i \\ 3 & 2 & -2 \\ 2-i & -2 & 2 \end{bmatrix}$. (b) Confirm its eigenvectors are mutually orthogonal. (c) Is this matrix symmetric or a Hermitian? Why? (d) Construct a unitary matrix U such that $U^\dagger H U = \Lambda$, where Λ is a real diagonal matrix. [24 points, each part is 6 points]

5. (a) Show following equations can be represented as the combination of a matrix and vectors [3 points]. (b) Find the equations have solutions only if $\eta = 1$ or 2 [9 points]

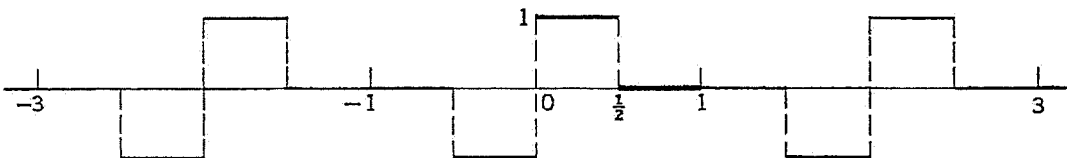
$$x + y + z = 1,$$

$$x + 2y + 4z = \eta,$$

$$x + 4y + 10z = \eta^2.$$

6. Evaluate $\oint (x \sin x - y)dx + (x - y^2)dy$ of a closed counterclockwise curve of a triangle with points (0,0), (1,1) and (0,2). [10 points]

7. For $f(x)$ as following



- (a) What is the periodicity λ of $f(x)$? [4 points]

- (b) What are coefficients a_0, a_n, b_n for $n = 1, 2, \dots$ if we write $f(x)$ in its Fourier Series [10 points]

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{2n\pi x}{\lambda} + b_n \sin \frac{2n\pi x}{\lambda}$$