國立成功大學

114學年度碩士班招生考試試題

編　號：38

系　所：物理學系

科　目：物理數學

日　期：0211

節　次：第 1 節

注　意：1.不可使用計算機
2.請於答案卷(卡)作答，於試題上作答，不予計分。

1. (a) Calculate the first three non-zero terms of the Maclaurin series of $\frac{1+x}{x}-\frac{1}{\sin x}$. [10 points]

   (b) Find $\lim_{x\to 0}\left(\frac{1+x}{x}-\frac{1}{\sin x}\right)$. [5 points]

2. Consider the matrix $M=\begin{pmatrix}5 & -2\\ -2 & 2\end{pmatrix}$.

   (a) Find the characteristic equation of M, and solve for the eigenvalues. [10 points]

   (b) Show that M satisfies its own characteristic equation. [5 points]

3. Consider the vector $\vec{V}=x^2\hat{i}+5x\hat{j}$.

   (a) Calculate $\oint \vec{V}\cdot d\vec{r}$ around the boundary of the square with vertices $(1,0)$, $(0,1)$, $(-1,0)$, $(0,-1)$ on the x-y plane counterclockwise. [10 points]

   (b) State Stokes' theorem, and use it to calculate the integration in (a). [10 points]

4. A rocket of (variable) mass m is propelled by steadily ejecting part of its mass at velocity u (constant with respect to the rocket). Gravity can be neglected.

   (a) The differential equation for the velocity v of the rocket is $m(dv/dm)=-u$ as long as $v \ll c$, where c is the speed of light. Find v as a function of m if $m=m_0$ when $v=0$. [5 points]

   (b) In the relativistic region (v/c not negligible), the rocket equation is $m\frac{dv}{dm}=-u\left(1-\frac{v^2}{c^2}\right)$. Solve this differential equation to find v as a function of m if $m=m_0$ when $v=0$. [10 points]

5. Given the function $f(x)=(\pi-x)/2$ on $(0,\pi)$.

   (a) Find the Fourier sine series of period 2π for $f(x)$. [10 points]

   (b) Calculate the average of $[f(x)]^2$ and use the result in (a) to evaluate $\sum_{n=1}^{\infty}\frac{1}{n^2}$. [10 points]

6. (a) Show that the poles in $\oint\frac{1+e^{inz}}{(z-1)^2(z+1)^2}dz$ are simple poles, and evaluate the contour integration around the upper half plane. [10 points]

   (b) Use the result in (a) to evaluate $\int_0^{\infty}\frac{\cos^2(\alpha\pi/2)}{\left(1-\alpha^2\right)^2}d\alpha$. [5 points]