(1) $\frac{12}{100}$ If $\vec{A} = (3x^2 + 6y, -14yz, 20xz^2)$, evaluate $\int \vec{A} \cdot d\vec{r}$ from $(0,0,0)$ to $(1,1,1)$ along the straight line connecting them.

(2) $\frac{12}{100}$ Find the solution of $y'' - 6y' + 8y = 16x + 12$ with the conditions $y(0) = y'(0) = 0$.

(3) $\frac{12}{100}$ Evaluate $\int_{-1}^{1} x^2 P_\ell(x) P_2(x) dx$ for any ℓ by using the recurrence relation of Legendre polynomials $(\ell+1)P_{\ell+1}(x)$
$$-(2\ell+1)x P_\ell(x) + \ell P_{\ell-1}(x) = 0$$

(4) $\frac{12}{100}$ Evaluate $\oint_{C_+} \frac{\sin z}{z^n} dz$ for $n = 0$ and any $\pm$ integers, where C is a unit circle $|z| = 1$.

(5) $\frac{12}{100}$ (a) Find the derivative of $f(z) = 3x^2 i$ at the points where it is differentiable.

(b) Where is the function analytic?

(6) $\frac{12}{100}$ (a) Find the Fourier series of a periodic function $f(x)$ with period $= T$, where
$$f(x) = \begin{cases} V_0 & |x| \le \frac{T}{6} \\ 0 & \frac{T}{6} < |x| \le \frac{T}{2} \end{cases}.$$

(b) Find the values to which the Fourier series converges for $x = 0, \frac{T}{6}, \frac{T}{2}$.

(7) $\frac{14}{100}$ Let $B = \begin{pmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{pmatrix}$.

(a) Find the inverse of B.

(b) Decompose B into the sum of a symmetric matrix S and an antisymmetric matrix A.

(8) $\frac{14}{100}$ What are the maximum and minimum values taken by the expression $Q = 5x^2 + 4y^2 + 4z^2 + 2xz + 2xy$ subject to $x^2 + y^2 + z^2 = 1$?

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