國立成功大學76學年度 物理研究所考試(古典力學試題) 共 1 頁 第 1 頁

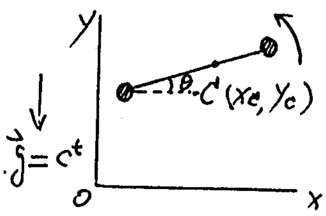
(20%)
1. Consisder the motion of a rigid system which consists of two masses m_1 and m_2 linked by a massless rigid rod. The center of mass is located at the point $C\,(x_c, y_c)$. Suppose that at $t=0$, m_1 is at the origin O and m_2 is rotating in the x-y plane with an angular velocity $\dot{\theta}_0$ in a constant gravitational field.

   (a) Find the position of C at any time t.

   (b) Find the total angular momentum and torque about O and C respectively.

   (c) Obtain the kinetic energy of the system.

   (d) What theorems do your results exemplify?



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2. Obtain the Differential Equation of Motion of a simple pendulum of length l and with a bob of mass m both by the Hamilton's principle and Euler-Lagrange's equation, a well as by D'Alembert's principle.

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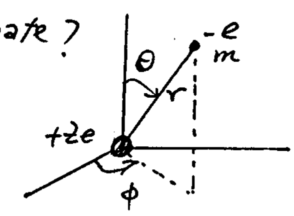
3. An electron of mass m and electric charge $-e$ moves in the field of a nucleus of charge $+Ze$, where Z is the atomic number. Choose for generalized coordinates the usual spherical coordinates, r, θ, ϕ as shown. Obtain the equation of motion by Hamiltonian formulation. Which one is the cyclic coordinate?



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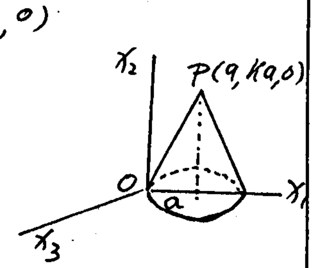
4. A uniform solid cone of mass M is placed on $x_1 x_3$-plane with its vertex at the point $P(a, ka, 0)$ and its base is the circle defined by equation

$$(x_1-a)^2 + x_3^2 = a^2 \quad \text{and} \quad x_2 = 0.$$

Determine the inertia tensor with respect to the origin O.



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5. A raindrop is falling from rest at height h. The damping force is linearly proportional to its instantaneous velocity $\vec{v}$. Find the expressions of the distance dropped and the times of falling for each of the following cases of approximation:

   (a) First approximation (b) Second approximation

   (c) Third approximation