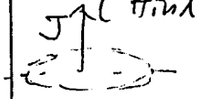
20% 1. (i) Consider a system of particles (with internal force), under what condition is (a) the linear momentum, (b) the angular momentum, of the system conserved?

(ii) Consider now two wheels each rotates with angular velocity w_1 and w_2. Bring these two into contact. (there is frictional force between them, but no external force) Will the angular momentum be conserved? why?

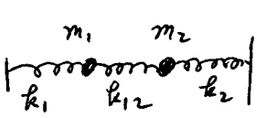
15% 2. A galaxy can be considered as a very large mass M of gas endowed initially with some angular velocity. The gas contracts under its gravitational interaction. The galaxy finally assumes a pancake shape. Explain this.

(Hint: consider the centrifugal potential energy)



20% 3. A hoop of mass m and radius R rolls without slipping down an inclined plane of mass M which makes an angle α with the horizontal. Find the Lagrange equations and the integrals of the motion for the case in which the plane can slide without friction along a horizontal surface.

20% 4. Consider two masses m_1, m_2 connected by three springs as shown in the figure.



Derive and discuss the motion of m_1 and m_2.

25% 5. A double Atwood machine is shown in the figure. Solve for the accelerations of the masses and the tensions in the strings. Use (a) Newton's law. (b) Lagrange equation and Lagrange multiplier method.

(The masses of the pulleys and friction are to be neglected)