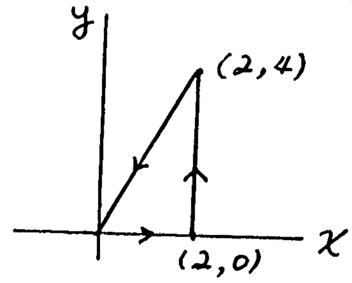
國立成功大學 77 學年度 物理研究所入學考試(物理數學 試題) 共 1 頁 第 頁

(1) Explain the Stokes' theorem and apply
10% it to evaluate the integral

$$\oint_C (2y\,\hat{i} + x\,\hat{j})\cdot d\bar{r},$$ where C

is shown in the figure.



(2) (a) Find the Fourier series of $f(x) = x^2$, $-\pi < x < \pi$
15% (b) From the series, find the value of $\sum_{n=1}^{\infty} \frac{1}{n^2}$

(3) Let $P_n(x)$ be the Legendre polynomial of degree n and
15% $r_m(x)$ be an arbitrary polynomial of degree m with $m < n$. Prove that $\int_{-1}^{1} r_m(x) P_n(x)\,dx = 0$

(4) Let $A = \begin{pmatrix} 3 & 0 & 1 \\ 0 & 2 & 0 \\ 5 & 0 & -1 \end{pmatrix}$. Find the eigenvalues of A^3 and A^{-1}!
15%

(5) Find the solution of $x^2 \frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 4y = 0$
15% if the initial conditions are $y(1) = 2$ and $y'(1) = 8$.

(6) Find the stationary functions $y(x)$ and $z(x)$, and the
15% stationary value of $I(y, z) = \int_0^1 [(y')^2 + (z')^2 + y'z']\,dx$,

with $y(0) = 1$, $y(1) = 0$, $z(0) = -1$, $z(1) = 2$.

(7) (a) Discuss the type of singular point of $f(z) = \sin\frac{1}{z}$ at $z = 0$.
15%
(b) Find the residue of $\sin\frac{1}{z}$ at $z = 0$.
(c) Evaluate $\oint_C \sin\frac{1}{z}\,dz$, $C : |z| = 1$.