(1) Evaluate $\iint_S x^2 dy dz + y^2 dz dx + z^2 dx dy$, where S is the boundary of the cube $0 \le x, y, z \le 1$.
14%

(2) Find the solution of $x\frac{dy}{dx} - y + x^3 = 0$; $y(1) = 0$
14%

(3) Evaluate $\int_{-1}^{1} P_n(x) P_m'(x) dx$, where P_n and P_m are any two Legendre polynomials, $P_m'(x) = \frac{d}{dx} P_m(x)$.
12%

(4) Use the generating function of Bessel functions $g(x,t) = e^{\frac{x}{2}(t - \frac{1}{t})}$ to prove the following recurrence relation $J_{n-1}(x) + J_{n+1}(x) = \frac{2n}{x} J_n(x)$
12%

(5) Evaluate $\int_0^\infty \frac{\cos kx}{x^2 + a^2} dx$, where $a > 0$, $K > 0$.
12%

(6) Let $L = \frac{d}{dx}$ be an operator acting on the vector space formed by all the polynomials of x with degree $\le n$. Find the matrix representation of L with the ~~[illegible]~~ basis $\{x^m, m = 0, 1, 2, \cdots, n\}$.
12%

(7) Show that if $AB = BA$ for every $n \times n$ matrix A, then $B = \alpha I_n$, where α is a constant, I_n a $n \times n$ identity matrix.
12%

(8) Please define a tensor.
12%