1.(15%) A particle is projected vertically upward from a point on the Earth's surface at a northern latitude λ. Show that it strikes the ground at a point $\frac{4}{3}\omega\cos\lambda\sqrt{8h^3/g}$ to the west(here ω is the angular velocity of the rotating earth). (Neglect air resistance and consider only small projection height h.)

2.(15%) Consider a particle of mass m moving in a plane and subject to an inverse square attractive force. Find the equations of motion and expressions for the general forces.

3.(30%) A particle of mass m moves under the influence a Yukawa-type potential

$$V(r) = \frac{ke^{-\alpha r}}{r}$$

where k and α are constants and $k < 0$.

(a) Find the force F(r) and make its plot.
(b) Find the effective potential $V_{eff}(r)$ and discuss the motion of the particle.
(c) Calculate the total energy E and angular momentum L if the particle moves in a circle of radius r_0.
(d) Calculate the time period of the circular motion and the period of small oscillations, that is, for slightly perturbed circular orbit.

4.(25%) Over a small temperature range, the tension ($\mathcal{F}$) of a stretched plastic rod is related to its length L and temperature T by

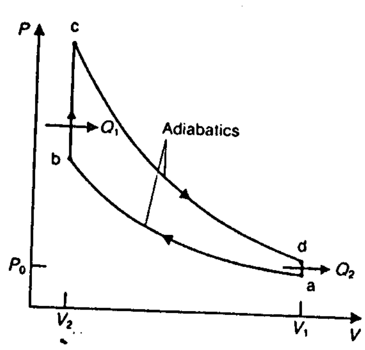
$$\mathcal{F} = a_0 T^2 (L - L_0)$$

where a_0 and L_0 are constants.

(a) Find $\left(\frac{\partial S}{\partial T}\right)_T$ for the rod.
(b) Knowing $S(T_0, L_0)$, find S(T, L) at any other lengths and temperatures.
(c) If the rod is stretched quasi-statically and adiabatically from T_0, L_0 to T_f, L_f. What is the final temperature of the rod?
(d) Find C_L when the length of the rod is L.

5.(15%) The Otto cycle is an approximation to the common gasoline engines. Let us assume that the working medium is just air which behaves likes an ideal gas rather than air and gasoline, with no chemical changes during the cycle. Also, instead of the heat being added internally by combustion, heat is assumed to be provided by the external heat reservoirs.

(a) Find the expression for the efficiency of the engine in terms of the compression ratio $r = V_2/V_1$ and $\gamma = C_P/C_V$.
(b) The preignition problem limits the value of the compression ratio r for a gasoline engine to 7 or 8, which gives a theoretical efficiency of ~54%. Try to explain why the real engines have much lower efficiency.



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