

(1) evaluate  $\int_{-\infty}^{\infty} e^{-x^2} dx$   
10%

(2) evaluate  $\int_{-\infty}^{\infty} e^{ix^2} dx$  ( $i^2 = -1$ )  
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(3) Solve  $\frac{d^2 y}{dx^2} - \frac{4}{x} \frac{dy}{dx} + \frac{4}{x^2} y = x$   
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(4) Solve  $f(x) = 1 - x + \int_0^x (x - \alpha) \cdot f(\alpha) d\alpha$   
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(5) Use the generating function  $g(x, t) = e^{-t^2 + 2tx}$   
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 $= \sum_0^{\infty} H_n(x) \frac{t^n}{n!}$ , to prove the relations

①  $H_{n+1} = 2xH_n - 2nH_{n-1}$     ②  $H'_n = 2nH_{n-1}$

(6) Find the eigenvalues and eigenvectors of  $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ .  
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(7) Define the Green's function and describe its applications.  
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Define the Tensor and describe its applications.