

1. (20%) Two thin-walled coaxial pipes of radii a and b ($a < b$) are lowered vertically into an oil bath. If a voltage V is applied between the pipes, what is the height that the oil rises? Assume the density and dielectric constant of the oil are ρ and K respectively.
2. (20%) (a) What is the current distribution $\vec{J}(x, y, z)$ to produce a magnetic field $\vec{B} = y^3 \hat{x}$? \hat{x} is the unit vector along x -axis.
 (b) The axial electric field intensity E_z in the accelerating tube of an ion accelerator is given by $E_z = E_0 + kz^2$, where E_0 and k are constants; z is measured from the center of the tube along its axis. The azimuthal component E_ϕ is zero. Find the radial electric field intensity E_r inside the tube, assuming a uniform charge density ρ within the tube. [$\nabla \cdot \vec{E} = \frac{1}{r} \frac{\partial}{\partial r}(rE_r) + \frac{1}{r} \frac{\partial E_\phi}{\partial \phi} + \frac{\partial E_z}{\partial z}$.]
3. (20%) A conducting sphere of radius a is charged to a potential V and spun about a diameter at an angular velocity ω . Find (a) the magnetic induction \vec{B} at the center of the sphere, (b) the magnetic dipole moment of the sphere.
4. (20%) (a) Find approximately the mutual inductance of two coaxial loops whose radii are a and b respectively. The planes of the loops are parallel to each other and are spaced at a distance c much larger than a and b . (b) If the current in the first loop is $I_1 = I_0 \sin \omega t$, find the electromotive force (emf) induced in the second loop.
5. (20%) (a) In a low-pressure (collisionless) plasma, find the phase velocity and group velocity, for $\omega > \omega_p$.
 (b) Calculate the skin depth and wave velocity in copper ($\sigma = 6 \times 10^7 \text{ mho/m}$) at a frequency of 1 GHz.