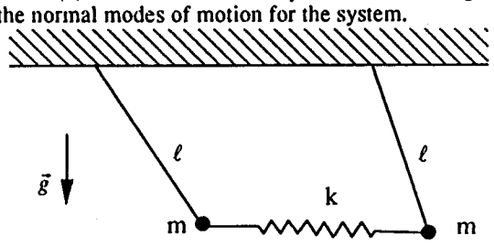
1.(25%) Consider a particle of mass m which is confined to move on the surface of a sphere, if the radius of the sphere is R and the influence of the gravitation can not be neglect.
(a) What is the equation of constraint of the the system?
(b) List the generalized coordinates and velocities needed to construct the configuration and phase spaces. Please indicate the dimensionality of the configuration and phase spaces.
(c) Find the Lagrangian equation of motion for the system.
(d) Find the force of constraint.
(e) Drive the canonical equations of motion for the system. Please give a very brief explanation on the meaning of each equation.
(f) Prove that the total energy of the system is conserved.

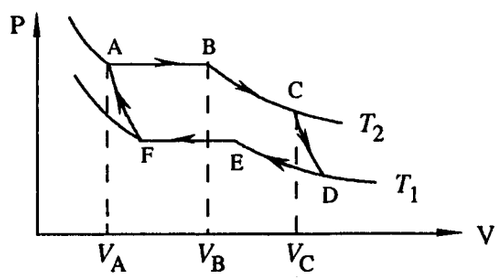
2.(25%) Two identical pendulums are suspended from the ceiling and coupled by means of a massless spring, of force constant k. Let us assume that, when both strings are vertical the spring is unstretched. The length of the string and the mass of the bob for each pendulum are ℓ and m, respectively. Find (a) the characteristic frequencies, (b) the eigenvectors, (c) the normal coordinates and (d) the normal modes of motion for the system.



3.(20%) When a particle with spin 1/2 is placed in a magnetic field H, its energy level is split into $-mH$ and $+mH$ and it has a magnetic moment m or $-m$ along the direction of magnetic field, respectively. Suppose a system of such particles is placed in a magnetic field H and is kept at temperature τ. Find(a) the partition function, (b) the Helmholtz free energy, (c) the entropy, (d) the total internal energy, and (e) the total magnetic moment of the system.

4.(16%) Consider a system that may be unoccupied with energy zero or occupied by one particle in either of two states one of energy zero and one of energy ε. Find
(a) the Gibbs sum of the system, (b) the thermal average occupancy of the system, (c) the thermal average occupancy of the state with energy ε, and (d) an expression for the thermal average energy of the system.

5.(14%) Two isotherms of 1 mole of a substance that can undergo a gas-liquid transition are shown in the accompanying P–V diagram. The absolute temperatures are T_2 and T_1, respectively. The substance is made to go through one cycle of a cyclic reversible transformation ABCDEF, as indicated in the diagram. The following information is given
(a) ABC and DEF are isothermal transformation.
(b) FA and CD are adiabatic transformation.
(c) In the gas phase (BCDE) the substance is an ideal gas. At A the substance is pure liquid.
(d) Latent heat along AB: L = 100 cal/mol.
T_2 = 200K, T_1 = 100K, V_A = 0.5 liter, V_B = 1 liter, and V_C = 1.5 liter.
Calculate the net amount of work done by the substance in one cycle.



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