(1) Evaluate $I = \int_{(1,0)}^{(-1,0)} (x^2 - y^2)\, dx$ along the semicircle
10% above the x-axis.

(2) Find the general solution of $\dfrac{d^2 y}{dx^2} + \dfrac{dy}{dx} + 2y = 4e^x + 2x^2$.
15%

(3) Find the eigenvalues and eigenfunctions of
15%
$$\frac{d^2 y}{dx^2} + \lambda y = 0$$

with $y(0) = y'(0)$, $y(\pi) = y'(\pi)$

(4) Evaluate $P_\ell(0)$ from the generating function of Legendre
15% polynomials or from the Rodrigues' formula.

(5) Find the Fourier sine series to represent the function
15% $y(x) = \pi - x$ in the range $0 \le x \le \pi$. What is
the sum of the series when (a) $x = \dfrac{\pi}{2}$, (b) $x = 0$?

(6) Find the Laurent series of $z^3 e^{\frac{1}{z}}$ at $z = 0$ and
15% determine the precise region of convergence. What is
the residue of $z^3 e^{\frac{1}{z}}$ at $z = 0$?

(7) Prove (a) the inverse of a $n \times n$ unitary matrix is
15% unitary.
(b) the product of two $n \times n$ unitary matrices
is also unitary.

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