(1) Evaluate the surface integral $\iint_S x^2 z \, dS$, where S is the upper half surface of a sphere of radius 1 and centered at the origin. (16%)

(2) Evaluate $\int_0^\infty \frac{dx}{x^4+1}$ by contour integration. (16%)

(3) If matrices A and B are related by a similarity transformation, i.e. A is similar to B, prove that
(a) A^n is similar to B^n.
(b) Determinants of A and B are equal. (16%)

(4) Following the Gram-Schmidt procedure, construct the lowest three polynomials $P_n(x)$ orthogonal with the inner-product defined by

$$(P_n, P_m) \equiv \int_0^1 P_n(x) P_m(x) \, dx$$

and normalized with $P_n(1) = 1$. (16%)

(5) (a) Find the Fourier series to represent the function

$$f(x) = \begin{cases} +1, & 0 < x < \pi \\ -1, & -\pi < x < 0 \end{cases}$$

(b) Find the value of the series $\sum_n |C_n|^2$ formed by squares of all the Fourier coefficients C_n. (18%)

(6) Determine the stationary function $y(x)$ and value of the following integral

$$I(y) = \int_0^{\frac{\pi}{4}} [(y')^2 + 2yy' - 16y^2] \, dx$$

with $y(0) = 0$, $y(\frac{\pi}{4}) = 1$. (18%)

034