1. (15%) Three identical metal spheres of radius a are placed at the corners of an equilateral triangle with sides $b \gg a$. Initially, all the spheres carry equal charge Q. They are then earthed in turn (one by one). What is the charge of each sphere at the end of the process ?

2. (15%) An infinitely long conducting wire in free space, carrying a current I, is parallel to an infinite plane surface of a semi-infinite iron block of permeability μ. The wire is at a distance D from the plane surface of iron. Calculate the force acting on the wire per unit of its length.

3. (15%) Two long coaxial solenoids has n turns per unit length and approximately equal radii a. (a) Find the mutual inductance when they overlap a distance x. (b) Find the force between them when currents I are flowing in each of them.

4. (15%) A variable capacitor consists of two thin coaxial cylinders of radii a and b, with $(b-a) \ll a$, free to move with respect to each other in the axial direction. A battery of terminal voltage V is connected across the cylinders of the capacitor. What is the force between the two cylinders when they are displaced a distance x with respect to each other ? (a) with the battery in place. (b) The battery is disconnected so that the charges on the cylinders are kept constant when they are displaced.

5. (20%) A long, thin conducting circular tube of radius a is split in two exact halves with the two spliting slits parallel to the axis of the tube. The two halves are kept at potentials V_0 and 0, respectively. Find the potential both inside and outside the tube.

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N. B., (i) $\int_0^{2\pi} \cos m\phi \cos n\phi \, d\phi = \pi \delta_{mn}$

(ii) $\int_0^{\pi/2} \cos m\phi \, d\phi = \begin{cases} \frac{\pi}{2}, & \text{for } m=0 \\ 0, & \text{for } m=2,4,6,\cdots \\ \frac{(-1)^{\frac{m-1}{2}}}{m}, & \text{for } m=1,3,5,\cdots \end{cases}$

6. (20%) The electric field of a plane wave propagating in the $+z$ direction in sea water ($\epsilon_r = 80$, $\mu_r = 1$, $\sigma = 4\ \Omega^{-1}\cdot m^{-1}$) is $\vec{E} = \hat{x}\, 100 \cos(10^7 \pi t)\ V\cdot m^{-1}$ at $z=0$. (a) Determine the attenuation constant, intrinsic impedance, phase velocity, wavelength, and skin depth. (b) Find the distance at which the amplitude of $\vec{E}$ is 1% of its value at $z=0$. (c) Write the expressions for $\vec{E}(z,t)$ and $\vec{H}(z,t)$.

N. B., $\ln 100 = 4.6$; $\epsilon_0 = 8.85\times10^{-12}\ C^2\cdot N^{-1}\cdot m^{-2}$;

$\mu_0 = 4\pi\times10^{-7}\ Wb\cdot A^{-1}\cdot m^{-1}$