(20%) 1. (a) $S = 1 + 2 + 4 + 8 + 16 + \cdots$

Then $2S = 2 + 4 + 8 + 16 + \cdots$

$\qquad = S - 1$

$\therefore S = -1$.

This is nonsense, why?

(b) Show that the series

$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \cdots$$

is diverge.

(20%) 2. Show that $\frac{d^2}{dx^2}$ is a hermitian operator, but $\frac{d}{dx}$ is not.

(20%) 3. The equation $4x^2 y'' + y = 0$ has a solution $\sqrt{x}$.

Find the other solution.

(20%) 4. (a) Evaluate $\displaystyle\int_{-\infty}^{\infty} \frac{\sin x}{x}\, dx$

(b) Using the integral $I = \displaystyle\int_{0}^{5} \frac{dx}{x-3}$ as an example, explain the meaning of "principle value".

(20%) 5. Consider the conic

$$5x^2 - 4xy + 2y^2 = 30.$$

Find the rotation matrix which express the equation of conic relative to its principle axes.

028