1. A conductor at potential $V = 0$ has the shape of an infinite plane except for a hemispherical bulge of radius $a$ (Fig. 1). A charge q is placed above the center of the bulge, a distance p from the plane ( or $p - a$ from the top of the bulge). What is the force on the charge ?   (15 points)

2. A cylindrical resistor (Fig. 2) has radius b, length L, and conductivity $\sigma_1$. At the center of the resistor is a defect consisting of a small sphere of radius a inside which the conductivity is $\sigma_2$. The input and output currents are distributed uniformly across the flat ends of the resistor.

(a) What is the resistance of the resistor if $\sigma_1 = \sigma_2$ ?

(b) Estimate the relative charge in the resistance to first order in $\sigma_1 - \sigma_2$ if $\sigma_1 \neq \sigma_2$. (Make any assumptions needed to simplify your method of estimation.)     (20 points)

3. A dipole of moment p is placed at a distance $x$ from a point charge q, so that p points directly toward q (Fig. 3). Find the force and the torque acting upon the dipole.     (15 points)

4. A coil of N turns is wrapped around an iron ring of radius d and cross section A ($d^2 \gg$ A). Assuming a constant permeability $\mu \gg 1$ for the iron:

(a) What is the magnetic flux $\phi = \int B_n \, dA$ as a function of current I ?

(b) If a gap of width $\delta$ ($\delta^2 \ll$ A) is cut in the ring, what is the flux for the same current I ?

(c) What is the field energy in the iron ? in the gap ?

(d) With such a gap in the ring, what is the self–inductance ?     (20 points)

5. An infinitely long conductive circular cylinder is split lengthwise and the two halves are held at potentials $U_0$ and $- U_0$ (Fig. 4). Find the potential everywhere inside.     (15 points)

6. Consider an elliptical polarized electromagnetic wave whose electric component is given by

$$E = i \, E_0 \sin \omega(t-z/c) + j \, E_0 \sin [\omega(t-z/c) + \pi/4].$$

Find the smallest and the largest value of the Poynting vector for this wave.     (15 points)

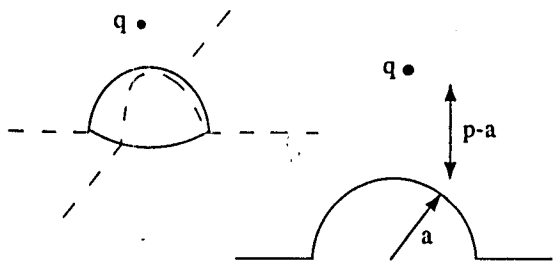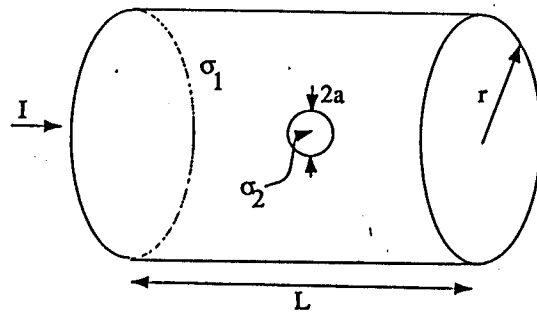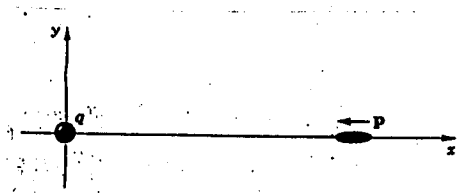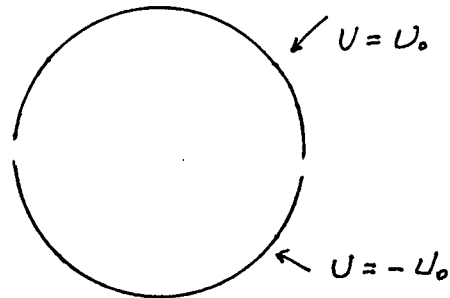Figure .1.



Figure .2.



Figure 3.



Figure 4.

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