(1) Find the force law for a central-force field that allows a particle to move in a spiral orbit given by $r = k\theta^2$, where k is a constant.
15%

(2) If a particle is fired due east from a point on the surface of the earth at a northern latitude λ with a velocity of magnitude V_0 and at an angle of inclination to the horizontal of α, show that the lateral deflection when the projectile strikes the earth is $d = \frac{4V_0^3}{g^2}\omega \sin\lambda \sin^2\alpha \cos\alpha$, where ω is the rotation frequency of the earth.
15%

(3). A massless spring of length b and spring constant k connects two particles of masses m_1 and m_2. The system rests on a smooth table and may oscillate and rotate.
20%

(a) Determine Lagrange's equations of motion.

(b) What are the generalized momenta associated with any cyclic coordinates?

(c). Determine Hamilton's equations of motion.

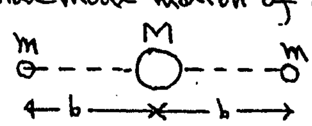
(4). A homogeneous cube of density ρ, mass M, and side of length b,
20%

(a). Calculate the inertia tensor of the cube, if one corner is at the origin, and the adjacent edges lie along the coordinate axes.

(b) Find principal moments of inertia and principal axes of the cube with the same origin.

(c) Find the transformation matrix which represents the transformation from the original axes to the principal axes.

(5) Determine the eigenfrequencies and describe the normal mode motion of a symmetrical linear triatomic molecule. (as shown)
20%



The central atom has mass M, the symmetrical atoms have masses m, and the elastic forces between atoms are represented by springs of force constant k. [Assume that only longitudinal vibrations are possible.)

(6) Show that the gravitational self-energy of a uniform sphere of mass M and radius R is $U = -\frac{3}{5}\frac{GM^2}{R}$.
10%