(1) Derive Euler's equations for motion in a force field, i.e.
10%. $(I_i - I_j)\omega_i \omega_j - \sum_k (I_k \dot{\omega}_k - N_k)\varepsilon_{ijk} = 0$

(2) Find the eigenfrequencies of the coupled circuits shown in Fig. 1.
15%

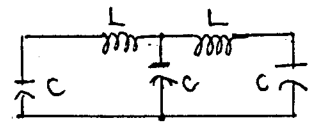

Fig. 1.

(3) A particle of mass m starts at rest on top of a smooth fixed hemi-
20% sphere of radius a. Find the force of constraint, and determine the angle at which the particle leaves the hemisphere. Try to solve the problem by using Newtonian method and Lagrange's method.

(4) Calculate the differential cross-section $\sigma(\theta)$ and the total cross-section σ_t
20% for the elastic scattering of a particle from an impenetrable sphere; the potential is given by

$$U(r) = \begin{cases} 0 & r > a \\ \infty & r < a, \end{cases}$$

where θ is the scattering angle in the frame of center of mass.

「Hint; $\Delta\Theta = \int_{r_{min}}^{r_{max}} \frac{(l/r^2)\,dr}{\sqrt{2\mu[E - U - (l^2/2\mu r^2)]}}$ 」

(5) Show that the angular deviation ε of a plumb line from the true vertical
15% at a point on the earth's surface at a latitude λ is

$$\varepsilon = \frac{r_0 \omega^2 \sin\lambda \cos\lambda}{g - r_0 \omega^2 \cos^2\lambda}$$

where r_0 is the radius of the earth, ω is the angular frequency of the earth's rotation.

(6) Find the force law for a central-force field that allows a particle to
20% move in a spiral orbit given by $r = k\theta^2$, where k is a constant.

「Hint; you may use the formula $\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r} = -\frac{\mu r^2}{l^2} F(r)$, but you have to derive it first. 」