

(1) To an observer in the rotating coordinates system, the effective force on a 20% particle is given by

$$\vec{F}_{\text{eff}} = m\vec{a}_r = \vec{F} - m\ddot{\vec{r}}_g - m\dot{\vec{\omega}} \times \vec{r} - m\vec{\omega} \times (\dot{\vec{r}}) - 2m\vec{\omega} \times \vec{v}_r$$

(a) Explain the physical meaning of each term in the equation. (10%)

(b) Try to verify the equation. (10%)

(2) Use Kepler's Laws of planetary motion to show that the gravitational force 20% must be central and that the radial dependence must be  $1/r^2$ .

(3) A particle of mass  $m$  is constrained to move on the inside surface of a 20% smooth cone of half-angle  $\alpha$ . The particle is subject to a gravitational force; (a) Determine a set of generalized coordinates and determine the constraints. (10%) (b) Find Lagrange's equations of motion. (10%)

(4) Determine the eigenfrequencies and describe the normal mode motion for 20% two pendula of equal length  $b$  and equal mass  $m$  connected by a spring of force constant  $k$ . The spring is unstretched in the equilibrium position.

(5) Discuss the force-free motion of a symmetric top. 20%

Hints; Following equations are useful in solving your problems.

(1).  $\frac{r}{r} = 1 + \epsilon \cos \theta$ .

(2).  $\frac{d^2}{d\theta^2} \left( \frac{1}{r} \right) + \frac{1}{r} = - \frac{mr^2}{L^2} F(r)$

(3).  $(I_i - I_j) \omega_i \omega_j - \sum_k (I_k \dot{\omega}_k - N_k) \epsilon_{ijk} = 0$